Comparative Advantage and Economies of Scale under Uncertainty*

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October 14, 2026

Abstract

We study how regional specialization patterns and welfare are affected by uncertainty and economies of scale in an open economy. We use a spatial equilibrium model with sectoral economies of scale, aggregate uncertainty, and irreversible mobility decisions made by heterogenous workers. We analytically characterize how the interactions between comparative advantage, both of workers and firms, economies of scale, and uncertainty shape specialization patterns and, hence, the gains from trade. We find empirical support for the model predictions by documenting the impact of aggregate changes in volatility of sectoral productivity on U.S. regional economies. We calibrate the model using detailed data on U.S. commuting zones and international trade, and extending hat-algebra methods to accommodate uncertainty. Quantitatively, we find that uncertainty shifts employment away from riskier sectors and locations, relative to a deterministic benchmark, lowering the U.S. aggregate gains from trade by around 20 percent. The interaction between uncertainty and scale economies brings down the benefits of specialization. For regions that would otherwise lose from trade, the presence of agglomeration forces and a more elastic labor supply mitigates those losses.

Key Words: trade; uncertainty; economies of scale; specialization
JEL Codes: F1; F2.

*Email: xiaoma@phbs.pku.edu.cn and nramondo@bu.edu. We thank Jingting Fan, Doireann Fitzgerald, Andrei Levchenko, Nitya Pandalai-Nayar, Andrés Rodríguez-Clare, Thomas Sargent, and participants at various conferences and seminars, for their helpful comments. We use Claude AI for editing, structuring proofs, and checking notation.

1 Introduction

Trade creates a fundamental tension between the efficiency gains from sectoral specialization and the insurance benefits of cross-country diversification. On the one hand, openness encourages countries and regions to specialize in the activities where they hold a comparative advantage, raising aggregate productivity and welfare. On the other hand, specialization can make economies more vulnerable to volatility by exposing them to sector- or country–specific shocks (Koren and Tenreyro 2007, 2013, Carvalho and Gabaix 2013). At the same time, international trade also provides access to diversified sources of supply and demand, which can mitigate domestic risk (Backus, Kehoe and Kydland 1992, Fitzgerald 2012, Caselli, Koren, Lisicky and Tenreyro 2020). This tension has crystalized in recent debates on the resilience of global value chains, and the vulnerability of specialized economies to aggregate shocks, such as the COVID-19 pandemic and the U.S. tariff war.

We study how uncertainty interacts with comparative advantage, economies of scale, and worker heterogeneity, to shape spatial specialization across and within countries, and hence, the gains from trade. We use the model to quantitatively analyze the type of globalization shocks studied by Autor, Dorn and Hanson (2013), and propose an extension of hat-algebra methods to calibrate the spatial model under uncertainty. Uncertainty shifts employment away from riskier sectors and locations, relative to a deterministic benchmark, lowering the U.S. aggregate gains from trade by around 20 percent. For regions that would otherwise lose from trade, the presence of agglomeration forces and a more elastic labor supply mitigates those losses.

We start by building a multi-sector multi-region multi-country model where uncertainty comes from a country-sector specific productivity shock, which affects all regions in a country. Sectors differ in their degree of economies of scale, and regions differ in their geography and fundamental productivities. The model has two periods: before and after uncertainty is realized. Irreversible decisions are made in terms of labor mobility: labor allocations are decided before uncertainty is realized by workers with heterogeneous productivities, modeled as a standard Roy-Fréchet (Lagakos and Waugh 2013). Production, consumption, and international trade happen once uncertainty is realized, and both factor and good prices adjust to clear all markets. Consumers are risk averse and face financial autarky, so that trade is balanced each period. In an extension, we allow for a fraction of workers in each country to trade a complete set of Arrow-Debreu (AD) securities before

uncertainty is realized, either in a global or in a national asset market.

The gains from trade, and with it, labor reallocation, are shaped by the interaction of comparative advantage, uncertainty, economies of scale, and the degree of workers' heterogeneity. With only the forces of comparative advantage, we recover the multi-sector ACR formula (Arkolakis, Costinot and Rodríguez-Clare 2012). If we add workers' heterogeneity, our gains from trade are as in Galle, Rodríguez-Clare and Yi (2023), while if we add economies of scale, we get an expression for the gains from trade similar to Kucheryavyy, Lyn and Rodríguez-Clare (2023). The inclusion of uncertainty adds a channel to the gains from trade relative to the models in the papers above.

Our analytical results, which we derive for the special case of frictionless trade and independent shocks across locations, show that labor tends to move away from riskier locations, and the effect is amplified the stronger the economies of scale and the higher the exposure to trade. These reallocation effects occur when there is a trade shock. In particular, a move from autarky can create loses, not gains, from trade for some locations: This occurs if labor moves to sectors with lower economies of scale. Worker heterogeneity could offset the effects as it creates less elastic responses of labor supply to shocks.

Before moving to our quantitative exercises, we test the predictions of the model using data for the United States on commuting zones (CZ) and industries, over the period 1990-2010. We document that CZs where industries became less riskier did not lose — or lost less — employment than riskier CZs. However, the negative effect of sectoral uncertainty on employment growth in an industry and CZ is driven by industries with strong economies of scale.

To calibrate the model, we use data on 26 countries, plus an aggregate for the rest of the world, and 20 sectors, spanning at least 20 years over the period 1970-2019. For the United States, we construct 721 CZs, following Autor et al. (2013), with 20 sectors each, and a time series that spans several decades all the way till 2017. For that year, we use the Commodity Flow Survey to construct sectoral trade flows between CZs, following procedures in the literature (e.g. Allen and Arkolakis 2014).

We first set some parameters externally, more saliently the sectoral trade elasticities and sectoral economies of scale, using estimates from Bartelme, Costinot, Donaldson and Rodríguez-Clare (2025), and the migration elasticity, from Fajgelbaum, Morales, Serrato and Zidar (2019).

We use a simulated method of moment procedure to jointly estimate the risk aversion parameter and the volatility of the country-industry shocks. To such end, we target the

responses of employment to uncertainty across U.S. CZs, given by our reduced-form estimates, and the country-sector standard deviation of the log of real value-added per worker observed in the data over 1970-2019.

Our procedure adapts hat-algebra methods (Dekle, Eaton and Kortum 2008) to accommodate uncertainty, exploiting the fact that employment shares are chosen ex-ante and hence do not vary with the realization of shocks.[1] In a first step, conditional on observed employment shares, we simulate the economy for many realizations of the shocks and compute equilibrium changes between each state and the average economy, which we pair with the observed cross-section. This yields the variance of growth of real value added per worker at the country-sector level, which we match to its empirical counterpart. In a second step, we calibrate risk aversion: holding volatility in the rest of the world fixed, we vary U.S. volatility to match its observed values in 1970–1990 and 1990–2010, compute the implied changes in region-sector employment shares, and run the same regression in the model as in the data.

Our quantitative results suggest that introducing uncertainty into a model with sectoral economies of scale substantially alters the size and distribution of the gains from trade. At the aggregate level, the United States gains about 1.87 percent from opening up to trade in the baseline model, while eliminating uncertainty raises gains by 20 percent to over 2.27 percent. Removing both uncertainty and scale effects yields a similar gain of 2.27 percent. These averages, however, mask large regional heterogeneity. Some CZs gain little from trade in the baseline calibration. Moreover, scale economies are key for ensuring positive gains from trade everywhere: without them, certain high-volatility regions experience losses. Employment reallocation responses mirror these patterns. From autarky to the baseline equilibrium, employment shifts away from high-volatility sectors and regions, with average losses of around 0.6 percent, but these changes decrease when uncertainty or scale economies are removed.

Insurance matters for these results. When workers can trade AD securities in a global market, uncertainty still lowers the gains from trade, since aggregate price risk cannot be diversified, but it no longer interacts with economies of scale. When AD markets are national, uncertainty has virtually no effect on the gains from trade, because sector-specific fluctuations largely offset each other at the country level. With partial access,

[1] Our method applies to static spatial models with an irreversible ex-ante decision, and pairs an observed cross-section with the average economy in the model. In contrast, the methods in Caliendo, Kortum and Parro (2025) and Kleinman, Liu and Redding (2025), suited to fully dynamic models, pair the observed economy with one realization of uncertainty.

uninsured workers gain less from trade than insured workers, and their reallocation away from volatile sectors also lowers the gains of insured workers through economies of scale.

Overall, the results highlight that uncertainty and scale effects jointly cut U.S. trade gains substantially, while also shaping the uneven regional distribution of those gains.

Literature Review. Our work is closest to research on multi-sector spatial models with economies of scale (Kucheryavyy et al. 2023), worker heterogeneity (Galle et al. 2023), and uncertainty (Caselli et al. 2020). Relative to the first two papers, our innovation is to introduce uncertainty together with an irreversible investment choice, a feature shared with several papers discussed below. Relative to Caselli et al. (2020), our distinct contribution is to embed the irreversible decision under uncertainty into a spatial model with sub-national geography, sectoral scale economies, and heterogeneous workers. The interaction of uncertainty and sectoral scale economies generates new effects, which are quantitatively relevant, while worker heterogeneity shapes the distributional implications of volatility across regions and industries.

A central element of our analysis is the treatment of trade under uncertainty in combination with sunk or irreversible factor allocations. Several models adopt this approach, including Caselli et al. (2020), Allen and Atkin (2022), Adamopoulos and Leibovici (2024), Kleinman et al. (2025), Castro-Vincenzi, Gracio, Khanna and Pandalai-Nayar (2026), and Fan and Luo (2025), among others.[2] These papers emphasize that producers must commit to factor allocations before uncertainty is realized, and therefore trade openness changes outcomes. For example, Allen and Atkin (2022) show that openness reduces the negative correlation between productivity and prices, with farmers adjusting ex-ante by shifting toward crops with less volatile yields. In their setting, as in ours, trade raises wage volatility, reduces price volatility, and generates ambiguous effects on real wage volatility. More broadly, this literature highlights the central tension that trade can increase volatility through specialization while reducing it through diversification—a point emphasized by Caselli et al. (2020), who, using a general equilibrium model based on Eaton and Kortum (2002) and Caliendo and Parro (2015), show that the net effect depends on the source of shocks.[3] A related point — that risks flattens the pattern of comparative advantage relative to a deterministic benchmark — is further emphasized by Castro-Vincenzi

[2] A related literature studies how volatility affects exporters' decisions, such as Handley and Limão (2017), Steinberg (2019), Alessandria, Arkolakis and Ruhl (2021), Alessandria, Choi and Ruhl (2021), Grossman, Helpman and Lhuillier (2023), and Alessandria, Khan, Khederlarian, Ruhl and Steinberg (2025).

[3] Caselli et al. (2020) show that when country specific shocks are dominant, in contrast to sector specific shocks, trade can decrease volatility.

et al. (2026) in the context of multi-stage sourcing decisions. While in their model the mechanism hinges on sourcing patterns across global value chains, in our model it comes from uncertainty interacting with economies of scale and worker heterogeneity across sub-national economies.

Our solution method builds on the "exact hat algebra" approach introduced by Dekle et al. (2008) and widely used in quantitative trade models. We extend this methodology to accommodate uncertainty by computing equilibrium changes across average counterfactual scenarios and across different realizations of stochastic shocks. In practice, this requires combining hat algebra methods with simulation techniques, generating a distribution of equilibrium outcomes for each realization of the shocks, which can be used to build moments to compare with data. In this sense, our work relates to recent advances in dynamic hat algebra methods in spatial economics (Caliendo, Dvorkin and Parro 2019, Caliendo et al. 2025, Kleinman et al. 2025, Fan and Luo 2025), which adapt the hat algebra framework to richer environments with dynamics, regional mobility, and aggregate shocks. Our approach further expands this toolkit.

Finally, our paper contributes to the trade literature on regional economies. Following Autor et al. (2013), many empirical studies have examined the effects of a large aggregate shock—most notably the "China shock"—on local labor markets (see Autor, Dorn and Hanson 2016, for a review). This work has inspired a growing quantitative literature integrating regional economies into international trade models to study the local impact of aggregate shocks (e.g., Caliendo et al. 2019, Caliendo, Parro, Rossi-Hansberg and Sarte 2018, Lyon and Waugh 2019, Galle et al. 2023, , among many others). Our contribution is to analyze how, in the presence of a one-time globalization shock, aggregate volatility shapes workers' regional and sectoral choices, and how this margin affects the gains from trade.

Our paper is structured as follows. Section 2 presents the baseline model, and Section 3 derives analytical results. Section 4 presents an extension of the baseline model to incorporate financial markets. We provide empirical evidence on the model's mechanisms in Section 5. Section 6 presents the model's calibration, followed by the quantitative analysis in Section 7. Finally, Section 8 concludes.

2 Model

In this section, we present a multi-sector spatial equilibrium model with aggregate uncertainty, economies of scale, and workers' region and sector choices. As a benchmark, we assume financial autarky and hand-to-mouth agents. In Section 4, we extend the model to incorporate Arrow-Debreu (AD) securities and agents with different access to them.

2.1 Preliminaries

The world consists of M regions, $m = 1, \ldots, M$, belonging to N countries, $i = 1, \ldots, N$. The number of regions belonging to country i is denoted by the set M_i, with $\cup_i M_i = M$. Each country is endowed with a measure L_i of workers. Workers are immobile across countries but mobile across regions of a country.

Each region has J sectors, $j = 1, \ldots, J$. Each sector is subject to a country-wide productivity shock, $A_{i,j}(s)$, where s is a possible realization of the shock, $s = \{1, 2, \ldots, S\}$, characterized by the probability $\mathbb{P}(s)$. Without loss of generality, we normalize the average shock to one for each country and sector, $\sum_s \mathbb{P}(s) A_{i,j}(s) = 1$, for all i, j, so that the variance is given by $\sigma^2_{i,j} = \sum_s \mathbb{P}(s) \left(A_{i,j}(s) - 1\right)^2$.

2.2 Workers

Workers make location and sector choices *before* uncertainty is realized. Once shocks are observed, they supply labor, produce, and consume. Workers are heterogeneous in their ability to work in different sectors and regions within a country i. The variable x_{mj} represents the efficiency units of labor in sector j of region m, is random, and drawn independently from a unit-Fréchet distribution with shape parameter $\kappa > 1$.

As in Caselli et al. (2020), we assume financial autarky and no savings technology, so that workers' real consumption in region m and sector j is given by their real income, $x_{mj} W_{mj}(s)/P_m(s)$, where $W_{mj}(s)$ is the wage rate per efficiency unit in region m, sector j, and state s, and $P_m(s)$ is the final-good price in region m and state s.

Workers have Constant-Relative-Risk-Aversion (CRRA) utility functions with relative risk-aversion coefficient $\gamma > 1$. For a worker in region m and sector j, the expected utility per

efficiency unit (i.e. $x_{mj} = 1$) is given by

$$U_{mj} = \frac{1}{1-\gamma} \sum_s \mathbb{P}(s) \left(\frac{W_{mj}(s)}{P_m(s)} \right)^{1-\gamma}. \tag{1}$$

In equilibrium, each worker chooses the region-sector pair that delivers the highest utility. With the Fréchet distributional assumption, the probability of a worker in country i choosing to locate and work region m and sector j is

$$\lambda_{mj} = \frac{U_{mj}^{\frac{\kappa}{1-\gamma}}}{\sum_{j'} \sum_{m' \in M_i} U_{m'j'}^{\frac{\kappa}{1-\gamma}}}, \tag{2}$$

or, equivalently, replacing U_{mj} for the expression in (1) yields

$$\lambda_{mj} = \frac{\left(\sum_s \mathbb{P}(s) \left(\frac{W_{mj}(s)}{P_m(s)} \right)^{1-\gamma} \right)^{\frac{\kappa}{1-\gamma}}}{\sum_{j'} \sum_{m' \in M_i} \left(\sum_s \mathbb{P}(s) \left(\frac{W_{m'j}(s)}{P_{m'}(s)} \right)^{1-\gamma} \right)^{\frac{\kappa}{1-\gamma}}}. \tag{3}$$

Total labor units in each region-sector pair are then given by

$$L_{mj} = C_1 L_i \lambda_{mj}^{1-\frac{1}{\kappa}}, \tag{4}$$

where $C_1 = \Gamma\left(1 - \frac{1}{\kappa}\right) > 0$, and $\Gamma(\cdot)$ is the Gamma function.

Workers cannot move after the shock is realized. But because the wage in local labor markets adjusts after uncertainty is realized, there is no unemployment in the model.

2.3 Final Goods

The final non-tradable good in region m is produced using goods from all sectors in the region,

$$Q_m(s) = \prod_j Q_{mj}(s)^{\alpha_j}, \tag{5}$$

where $\alpha_j \in [0,1]$ is the share of expenditure in intermediate goods from sector j, and $\sum_j \alpha_j = 1$. The price index for the final good in region m is

$$P_m(s) = \prod_j \left(\frac{P_{mj}(s)}{\alpha_j} \right)^{\alpha_j}, \tag{6}$$

where $P_{mj}(s)$ is the price index for the intermediate good from sector j.

2.4 Intermediate Goods

Each sector j, region $m \in M_i$, produces a non-tradable intermediate good under perfect competition. The sectoral intermediate good is composed of a continuum of tradable varieties $\omega \in [0,1]$. The production technology is Constant-Elasticity-of-Substitution (CES) with elasticity of substitution $\eta_j > 1$,

$$Q_{mj}(s) = \left(\int_0^1 q_{mj}(\omega, s)^{\frac{\eta_j - 1}{\eta_j}} d\omega \right)^{\frac{\eta_j}{\eta_j - 1}}, \tag{7}$$

where $q_{mj}(\omega, s)$ is the quantity demanded of variety ω in state s.

Production of variety ω belonging to sector j in region m is given by

$$q_{mj}(\omega, s) = A_{mj}(s) z_m(\omega) l_{mj}(\omega, s), \tag{8}$$

where $q_{mj}(\omega, s)$ is output of variety ω in region m, sector j, and state s, $A_{mj}(s)$ is aggregate productivity of sector j in region m and state s, $z_m(\omega)$ is variety-level productivity, and $l_{mj}(\omega, s)$ is the number of efficiency units used in production.

We make the following assumptions. First, $z_m(\omega)$ is randomly drawn independently across regions and varieties from a unit- Fréchet distribution with shape parameter $\theta_j > 0$. Second, aggregate productivity is given by

$$A_{mj}(s) \equiv \bar{A}_{mj} A_{i,j}(s), \tag{9}$$

where $\bar{A}_{mj}$ is the fundamental productivity of sector j in region m, and $A_{i,j}(s)$ is the country-sector specific shock. Hence, the volatility of a regional-industry shock is just given by $v_{mj} = \bar{A}_{mj} \sigma_{i,j}$.[4]

[4] In an extension in Section 7.5, we introduce region-sector specific shocks.

We introduce economies of scale through $\bar{A}_{mj}$, as specified next.

Assumption 1 (Economies of scale). $\bar{A}_{mj} = C_{mj} L_{mj}^{\rho_j}$ *where* $\rho_j \geq 0$.

Fundamental region-sector productivity is log-proportional to the amount of labor in that region and sector, L_{mj}. The parameter ρ_j governs the degree of economies of scale in the sector. When $\rho_j = 0$ there is no economies of scale and fundamental productivity is not endogenous to the amount of labor allocated to industry j. When $\rho_j > 0$ productivity increases with the amount of labor in the industry, reflecting industry-level external economies of scale. The region-sector fixed effect, C_{mj}, captures any other factor affecting region-sector fundamental productivity.

Finally, we assume iceberg-type trade costs from region l to m, $\tau_{lm}^j \geq 1$, with $\tau_{mm}^j = 1$.

Producers of the composite intermediate good in sector j and region m source each variety ω from the cheapest world-wide supplier,

$$p_{mj}(\omega, s) = \min_{l \in M} \frac{\tau_{lm}^j W_{lj}(s)}{A_{lj}(s) z_l(\omega)}. \tag{10}$$

The share of expenditure in sector j by region m devoted to goods from region $l \in i$, in state s, is given by

$$\pi_{lm,i}^j(s) = \frac{(\tau_{lm}^j W_{lj}(s)/A_{i,j}(s))^{-\theta_j} \tilde{C}_{lj} \lambda_{lj}^{\tilde{\rho}_j} L_i^{\rho_j \theta_j}}{\sum_{i'} \sum_{l' \in M_{i'}} (\tau_{l'm}^j W_{l'j}(s)/A_{i',j}(s))^{-\theta_j} \tilde{C}_{l'j} \lambda_{l'j}^{\tilde{\rho}_j} L_{i'}^{\rho_j \theta_j}}. \tag{11}$$

where $\tilde{C}_{lj} \equiv C_{lj}^{\theta_j} C_1^{\rho_j \theta_j}$.

The price index for the sector j's intermediate good is

$$P_{lj,i}(s) = C_j \left(\sum_{i'} \sum_{m \in M_{i'}} \left(\tau_{ml}^j \frac{W_{mj}(s)}{A_{i',j}(s)} \right)^{-\theta_j} \tilde{C}_{mj} \lambda_{mj}^{\tilde{\rho}_j} L_{i'}^{\rho_j \theta_j} \right)^{-\frac{1}{\theta_j}}, \tag{12}$$

with $C_j \equiv \Gamma\left(1 - \frac{\eta_j - 1}{\theta_j}\right)^{\frac{1}{1-\eta_j}}$ and $1 + \theta_j - \eta_j > 0$, for all j.

2.5 Equilibrium

We can solve the model's equilibrium backwards. Once aggregate uncertainty is realized, given the region-sector choices of workers, the competitive equilibrium consists of

a collection of state-dependent wages, prices, and quantities such that in each state s: (1) workers maximize their utility given prices and income; (2) firms maximize profits taking wages and prices as given; and (3) goods' and labor markets clear, while trade is balanced. Before uncertainty is realized, workers make location-sector decisions to maximize their expected utility.

We define effective economies of scale as

$$\tilde{\rho}_j \equiv \frac{\kappa - 1}{\kappa} \rho_j \theta_j, \tag{13}$$

and proceed to state the following assumption.

Assumption 2. *For all j,*

$$\tilde{\rho}_j - \frac{\theta_j}{\kappa} \in [0, 1).$$

Kucheryavyy et al. (2023) show that in the deterministic model with heterogeneous workers the condition above entails that, given wages, labor allocations exist, are unique, each sector (and region) has positive employment, and the aggregate gains from trade are positive. If trade is frictionless, or each region is sufficiently small (i.e. small open economy assumption), they can also show that equilibrium wages are unique.

3 Analytical Results

In this section, we analyze the effects of uncertainty and economies of scale on regional and sectoral specialization. Additionally, we derive the gains from trade. All proofs are in Appendix A.

We denote by: $\bar{X}$ the expected value of $X(s)$ across states s, $\bar{X} \equiv \sum_s \mathbb{P}(s) X(s)$; X^* the value of $X(s)$ in the deterministic equilibrium; $\hat{X}(s) \equiv dX(s)/X^*$ deviations around X^* for state-dependent variables ($\hat{X} \equiv dX/X^*$ for non-state dependent variables); and $\hat{\bar{X}} \equiv \sum_s \mathbb{P}(s) \hat{X}(s)$. We normalize global expenditure in each state s to one.

3.1 The Effects of Uncertainty: Autarky vs Frictionless Trade

We start by comparing the economy under autarky and the open economy. We first calculate the volatility of real wages. We derive the results under frictionless trade across regions and countries so that prices are equalized worldwide in each state s.

Lemma 1 (Volatility of the real wage). *Assume that productivity shocks are independently distributed, and that trade is frictionless across regions worldwide, $\tau^j_{mk} = 1$ for all $m \neq k$. The volatility of the real wage in country i and sector j is*

$$\Xi_{i,j} \equiv var\left(\hat{W}_{i,j}(s) - \hat{P}_i(s)\right) = \left(\frac{\theta_j}{1+\theta_j}(1-\bar{\pi}_{i,j}) + \alpha_j\bar{\pi}_{i,j}\right)^2 \sigma^2_{i,j} + \left(\frac{\theta_j}{1+\theta_j} - \alpha_j\right)^2 \sum_{i'\neq i}(\bar{\pi}_{i',j})^2\sigma^2_{i',j} + \sum_{k\neq j}\sum_{i'}(\bar{\pi}_{i',k})^2\alpha_k^2\sigma^2_{i',k} \tag{14}$$

where $\bar{\pi}_{i,j} \equiv \sum_{m\in M_i}\bar{\pi}_{mj}$, is the expected share of expenditure in each region of country i and sector j, from all other regions, including self, in country i. Under autarky, $\bar{\pi}_{i,j} = 1$ for all j, and (14) *collapses to*

$$\Xi^A_{i,j} \equiv var\left(\hat{W}^A_{i,j}(s) - \hat{P}^A_i(s)\right) = \sum_k \alpha_k^2\sigma^2_{i,k} = \Xi^A_i. \tag{15}$$

The lemma shows that the effect of trade openness on real wage volatility is ambiguous. Under autarky, wages are constant across states, $W^A_{i,j} = \alpha_j/L^A_{i,j}$, so that the only source of volatility in real wages is due to prices. Hence, while trade openness unambiguously increases wage volatility relative to autarky—and the effect is larger, the weaker the effect of comparative advantage (higher θ_j) —its effect on price volatility is ambiguous.

The offsetting effects of trade openness on wages and prices have been emphasized by Caselli et al. (2020), who show that the impact of trade on volatility critically depends on the structure of shocks—specifically, whether they are sector- or country-specific. In line with this reasoning, when shocks are purely sectoral and common across countries (i.e., $\sigma_{i,j} = \sigma_j$ for all i, j), inspecting (14) reveals that trade openness reduces price volatility by diversifying expenditure across sourcing locations, while wage responses reflect the reallocation of production across sectors. Consistent with this mechanism, Allen and Atkin (2022) show in the context of agricultural markets that openness increases wage volatility, reduces price volatility, and has an ambiguous net effect on real wage volatility.

A clear condition can be found if we assume that countries are identical (except for the realization of shocks). In that case, (14) becomes

$$\Xi_j = \sigma^2_j\frac{\theta_j^2}{(1+\theta_j)^2}\frac{N-1}{N} + \frac{1}{N}\sum_k \alpha_k^2\sigma_k^2. \tag{16}$$

If $1/\theta_j > (1-\alpha_j)/\alpha_j$, then real-wage volatility in sector j would be lower in the open economy. While nominal wage volatility is always larger in the open economy, price

volatility is always lower by a factor of N in the open economy. The condition is sufficient to ensure that the price effect is larger than the wage effect.

What happens with ex-ante labor allocations? Using (2) and a second order approximation of the expected utility in (1) yields

$$\lambda_{i,j} = \left(\frac{\bar{W}_{i,j}}{\bar{P}_i}\right)^{\kappa} \frac{\left(1 - \frac{\gamma(1-\gamma)}{2}\Xi_{i,j}\right)^{\frac{\kappa}{1-\gamma}}}{\sum_{j'}\left(\frac{\bar{W}_{i,j'}}{\bar{P}_i}\right)^{\kappa}\left(1 - \frac{\gamma(1-\gamma)}{2}\Xi_{i,j'}\right)^{\frac{\kappa}{1-\gamma}}}. \tag{17}$$

Further assuming symmetric countries, we can solve for the labor share in the open economy,

$$\lambda_j = \frac{\alpha_j \mathbb{A}_j \left(1 - \frac{\gamma(1-\gamma)}{2}\Xi_j\right)^{\frac{1}{1-\gamma}}}{\sum_k \alpha_k \mathbb{A}_k \left(1 - \frac{\gamma(1-\gamma)}{2}\Xi_k\right)^{\frac{1}{1-\gamma}}},$$

where $\mathbb{A}_j \equiv \sum_s \mathbb{P}(s) \frac{A_{i,j}(s)^{\frac{\theta_j}{1+\theta_j}}}{\sum_{i'} A_{i',j}(s)^{\frac{\theta_j}{1+\theta_j}}}$, and Ξ_j is given by (16).[5] It is immediate to see that with no uncertainty, the labor share in a sector is pinned down by the expenditure share, $\lambda_j^* = \alpha_j$. Under the same conditions of Lemma 1, this is also the autarkic labor share in sector j, $\lambda_j^A = \alpha_j$.

Hence, with uncertainty, sectors with low volatility in real wages (low Ξ_j) have employment shares above the autarky levels, as well as their deterministic levels, $\lambda_j > \alpha_j$, while the opposite is true for sectors with high real wage volatility.

Next, we dive deeper into the effects of uncertainty on labor reallocation, as well as its interaction with economies of scale.

3.2 Economies of scale, uncertainty, and sectoral reallocation

Uncertainty affects ex-ante workers' decisions on which sector and region to work. It also changes the effects of trade openness on specialization patterns across sectors with different degrees of scale economies. In turn, these effects will shape the gains from trade. We consider a country i, which is small enough to not affect foreign wages and prices.

Proposition 1 (**Employment reallocation**). *For a small country i, suppose that shocks are independent across industries and countries. Assume that trade is frictionless across regions world-*

[5] In this case, expected prices $\bar{P}$, which are equalized across regions worldwide, cancel out and the expected wage is $\bar{W}_j = \alpha_j C_1^{-1} (\lambda_j)^{\frac{1}{\kappa}-1} L^{-1} \mathbb{A}_j$.

wide, $\tau_{mk}^{j} = 1$ for all $m \neq k$. Then relative to the deterministic labor shares:
(1) Sector j's employment in region m decreases the higher the industry level volatility,

$$\frac{\partial \lambda_{mj}/\lambda_{mj}^{*}}{\partial \sigma_{i,j}^{2}} < 0;$$

(2) The effect is amplified with stronger industry-level economies of scale,

$$\frac{\partial^2 \lambda_{mj}/\lambda_{mj}^{*}}{\partial \sigma_{ij}^{2} \partial \rho_{j}} < 0,$$

higher relative risk aversion,

$$\frac{\partial^2 \lambda_{mj}/\lambda_{mj}^{*}}{\partial \sigma_{i,j}^{2} \partial \gamma} < 0,$$

and if $\frac{1}{\theta_j} < \frac{1-\alpha_j}{\alpha_j}$, higher country-level exposure to trade (i.e. lower $\pi_{i,j}^{*}$),

$$\frac{\partial^2 \lambda_{mj}/\lambda_{mj}^{*}}{\partial \sigma_{i,j}^{2} \partial \pi_{i,j}^{*}} > 0,$$

with $\pi_{i,j}^{*} = \sum_{m \in M_i} \pi_{mj}^{*}$.

Ex-ante, workers avoid sectors with higher shock volatility, which are sectors with higher volatility in real wages (Lemma 1), and reallocate towards less-volatile sectors, relative to the deterministic equilibrium. This effect is similar to the one in Allen and Atkin (2022) in the context of farmers choosing less volatile crops to farm.

The reallocation towards less volatile sectors is stronger in sectors with stronger economies of scale, or if workers are more risk averse. Each worker foresees that higher volatility will drive other workers away from the sector, hence, weakening the effect of scale.

Finally, opening up to trade reinforces reallocation away from volatile sectors as long as specialization forces are weak relative to expenditure diversification — a case that tends to increase the volatility of real wages in the open economy.

3.3 The gains from trade

We now characterize gains from trade and show how they depend on uncertainty, economies of scale, and worker heterogeneity. We define the gains from trade as the ratio of the average expected utility between the current equilibrium and the counterfactual equilibrium with no trade ($\tau_{mk}^{j} \to \infty$, for all $m \in M_i$ $k \in M_{i'}$, $i \neq i'$), across workers with different

efficiencies in region m and industry j.

Average expected utility is equalized across regions and sectors within a country due to selection, so that

$$\mathbb{U}_{mj} = \mathbb{U}_i \qquad \text{for all } m \in M_i,\ j.$$

Moreover,

$$\mathbb{U}_i = \Gamma\left(\frac{\kappa+\gamma-1}{\kappa}\right)\left(\sum_{m\in M_i}\sum_j U_{mj}^{\frac{\kappa}{1-\gamma}}\right)^{\frac{1-\gamma}{\kappa}}, \tag{18}$$

where $\kappa+\gamma-1>0$ –see Appendix A.4 for derivations. Therefore, defining gains from trade relative to autarky as

$$GT_i \equiv \left(\frac{\mathbb{U}_i}{\mathbb{U}_i^A}\right)^{\frac{1}{1-\gamma}}, \tag{19}$$

a second-order approximation of U_{mj} yields

$$\left(\frac{U_{mj}}{U_{mj}^A}\right)^{\frac{1}{1-\gamma}} \approx \underbrace{\frac{\bar{W}_{mj}/\bar{P}_m}{\bar{W}_{mj}^A/\bar{P}_m^A}}_{\text{Change in average real wage}} \times \underbrace{\left(\frac{1-\frac{(1-\gamma)\gamma}{2}\Xi_{mj}}{1-\frac{(1-\gamma)\gamma}{2}\Xi_{mj}^A}\right)^{\frac{1}{1-\gamma}}}_{\text{Change in volatility of real wage}}, \tag{20}$$

where $\Xi_{mj} \equiv \mathrm{Var}(\hat{W}_{mj}(s) - \hat{P}_m(s))$.

We can further decompose the change in the average real wage as

$$\frac{\bar{W}_{mj}/\bar{P}_m}{\bar{W}_{mj}^A/\bar{P}_m^A} = \underbrace{\frac{\bar{W}_{mj}/\prod_j \bar{W}_{mj}^{\alpha_j}}{\bar{W}_{mj}^A/\prod_j \bar{W}_{mj}^{A\alpha_j}}}_{\text{Workers heterogeneity}} \times \underbrace{\prod_j\left(\frac{\bar{\pi}_{mm}^j}{\bar{\pi}_{mm}^{Aj}}\right)^{-\frac{\alpha_j}{\theta_j}}}_{\text{Multi-sector ACR}} \times \underbrace{\prod_j\left(\frac{\lambda_{mj}}{\lambda_{mj}^A}\right)^{\alpha_j\rho_j}}_{\text{Economies of scale}} \times \underbrace{\prod_j\left(\frac{\lambda_{mj}}{\lambda_{mj}^A}\right)^{-\frac{\alpha_j\rho_j}{\kappa}}}_{\text{Interaction: ES and heterogeneity}}. \tag{21}$$

The first term on the right-hand side captures changes in relative wages when labor is imperfectly mobile (see Arkolakis et al. 2012, Galle et al. 2023), while the second represents the multi-sector version of the standard ACR formula. The third term appears because of the presence of economies of scale, as in Kucheryavyy et al. (2023), and the fourth term captures the interaction between this force and worker heterogeneity.

To make a sharp statement on the effects of uncertainty, economies of scale, and worker heterogeneity on the gains from trade, the next proposition specifies the formula in the case of frictionless trade and i.i.d. shocks.

Proposition 2 (**The gains from trade**). *Assume that shocks are independent across industries*

and countries, and trade is frictionless across and within countries. The gains from trade are given by

$$GT_i \approx \prod_j \left(\frac{\lambda_{i,j}}{\lambda_{i,j}^A}\right)^{-\frac{\alpha_j}{\kappa}} \prod_j \left(\bar{\pi}_{i,i}^j\right)^{-\frac{\alpha_j}{\theta_j}} \prod_j \left(\frac{\lambda_{i,j}}{\lambda_{i,j}^A}\right)^{\alpha_j\rho_j} \prod_j \left(\frac{\lambda_{i,j}}{\lambda_{i,j}^A}\right)^{-\frac{\alpha_j\rho_j}{\kappa}} \prod_j \left(\frac{1-\frac{(1-\gamma)\gamma}{2}\Xi_{i,j}}{1-\frac{(1-\gamma)\gamma}{2}\Xi_{i,j}^A}\right)^{\frac{\alpha_j}{1-\gamma}}, \tag{22}$$

where $\lambda_{i,j}^A = \alpha_j$*, and* $\Xi_{i,j}, \Xi_{i,j}^A$ *are given by* (14) *and* (15)*, respectively.*

With no uncertainty, $\Xi_{i,j}^A = \Xi_{i,j} = 0$, and no worker heterogeneity, $\kappa \to \infty$, the gains from trade would collapse to the second and third term in (22), which is the same expression as in Kucheryavyy et al. (2023).[6] Conditional on $\bar{\pi}_{ii}^j$, the gains from trade are higher when labor is allocated to sectors with higher economies of scale, $\sum_j \alpha_j\rho_j \log(\lambda_{i,j}/\alpha_j) > 0$. In turn, with no uncertainty, $\Xi_{i,j}^A = \Xi_{i,j} = 0$, and no economies of scale, $\rho_j = 0$ for all j, only the first and second term in (22) survive. We then recover the expression for the gains from trade in Galle et al. (2023), which feature a model with worker heterogeneity, but neither scale economies nor uncertainty. In this case, conditional on $\bar{\pi}_{ii}^j$, the gains from trade are higher when labor shares moves away from sectors with high expenditure shares, $-(1/\kappa)\sum_j \alpha_j \log(\lambda_{i,j}/\alpha_j) > 0$. The effect is stronger, the less elastic labor supply is (i.e. lower κ). When worker heterogeneity is coupled with economies of scale, there is an additional effect on the gains from trade captured by the fourth term in (22). Here, gains from trade increase as long as $-(1/\kappa)\sum_j \rho_j\alpha_j \log(\lambda_{i,j}/\alpha_j) > 0$, meaning that labor moves away from sectors with high economies of scale. On net, $(1-1/\kappa)\sum_j \rho_j\alpha_j \log(\lambda_{i,j}/\alpha_j) > 0$, with worker heterogeneity offsetting the effect of economies of scale on sectors that gain employment; in the extreme, if workers were virtually immobile (i.e. $\kappa \to 1$), scale effects would be completely neutralized. Which are the sectors that gains employment? In an economy with uncertainty, these are the sectors for which the volatility of real wages decreases with openness. As we show in Section 3.1, this can happen unambiguously as long as countries are identical and $(1-\alpha_j)/\alpha_j < 1/\theta_j$.

Once we condition on $\lambda_{i,j}$ and $\bar{\pi}_{ii}^j$, there is no interaction between scale economies and uncertainty. However, we know that $\lambda_{i,j}$ decreases from its autarky levels with volatility. Hence, conditional only on $\bar{\pi}_{ii}^j$, the gains from trade decrease with real-wage volatility, but this effect gets offset if those sectors are the ones with weak scale economies.

Losses from trade are possible due to reallocation of labor towards sectors with weak

[6]The online appendix of Kucheryavyy et al. (2023) presents a version of their baseline model with worker heterogeneity. Their goal, however, is to show conditions under which a multi-sector model of trade with scale economies delivers a unique and interior solution for employment allocations.

economies of scale from sectors with increased real-wage volatility.

Finally, the last term in (22) captures the additional channel affecting the gains from trade due of the presence of risk-averse workers. Again, once we condition on sectoral labor shares and domestic trade shares, the effect of volatility on these gains hinge on whether real wage volatility increases or decreases with openness.

These analytical results, derived under special conditions, shed light on the mechanics of the model and guide the understanding of counterfactual exercises performed with the general version of the model. Next, we first document data patterns that are qualitatively in line with the model's predictions. We then calibrate a richer version of the model (one with input-output linkages) and use it for counterfactual analysis.

4 Arrow-Debreu Securities and Partial Access

The baseline model assumes financial autarky: trade is balanced within each state s, so that workers cannot use financial markets to reallocate resources across states. We now relax this assumption and introduce Arrow-Debreu (AD) securities, which allow workers to insure against country-sector aggregate shocks. We assume that a fraction $\mu_i \in [0, 1]$ of workers in country i have access to those securities, while a fraction $1 - \mu_i$ have *no access* and face financial autarky. When $\mu_i = 1$ for all i we are in the full insurance case, while when $\mu_i = 0$ for all i we nest the case of financial autarky in our baseline. We treat μ_i as *exogenous*, but it can be easily micro-founded (e.g. through bank access).

Before uncertainty is realized—and after workers have chosen their region-sector pairs—a frictionless asset market with a complete set of AD securities opens for the fraction μ_i of workers. The security corresponding to state s pays one unit of the numeraire if state s occurs and zero otherwise. We denote the price of this security in country i by $Q_i(s)$. Under a globally integrated asset market, these prices are equalized across countries, so that $Q_i(s) \equiv Q(s)$ for all i. In contrast, when asset markets are segmented across countries, but fully integrated within each country, security prices may differ across countries, and $Q_i(s)$ reflects the valuation of state-contingent claims by workers in country i.

4.1 Setup

Workers without access to AD securities consume all their wage income in each state, that is, they are hand-to-mouth, so that $C_{mj}^{H}(s) = W_{mj}(s)$, as in our baseline model. Workers with access to AD securities choose a state-contingent consumption plan $C_{mj}^{AD}(s)$ to solve

$$\max_{\{C_{mj}^{AD}(s)\}} \sum_{s} \mathbb{P}(s) \left(\frac{C_{mj}^{AD}(s)}{P_m(s)} \right)^{1-\gamma} \tag{23}$$

$$\text{s.t.} \quad \sum_{s} Q_i(s) C_{mj}^{AD}(s) = \sum_{s} Q_i(s) W_{mj}(s).$$

The first-order condition implies

$$\frac{C_{mj}^{AD}(s)}{C_{mj}^{AD}(s')} = \left(\frac{P_m(s)}{P_m(s')} \right)^{\frac{\gamma-1}{\gamma}} \left(\frac{Q_i(s')/\mathbb{P}(s')}{Q_i(s)/\mathbb{P}(s)} \right)^{\frac{1}{\gamma}}. \tag{24}$$

To ease notation, we assume that all states occur with equal probability, $\mathbb{P}(s) = \mathbb{P}(s')$; a state with a higher probability of occurrence can equivalently be represented by multiple states with identical outcomes and equal probabilities. Combining the first-order condition with the budget constraint, we obtain

$$C_{mj}^{AD}(s) = \zeta_{mj}\, Q_i(s)^{-\frac{1}{\gamma}} P_m(s)^{\frac{\gamma-1}{\gamma}}, \tag{25}$$

where ζ_{mj} is a state-independent constant equal to $\frac{\sum_{s'} Q_i(s') W_{mj}(s')}{\sum_{s'} Q_i(s')^{\frac{\gamma-1}{\gamma}} P_m(s')^{\frac{\gamma-1}{\gamma}}}$.

The expected utility per efficiency unit, by financial-access type, is

$$U_{mj}^{AD} = \frac{1}{1-\gamma} \sum_{s} \mathbb{P}(s) \left(\frac{C_{mj}^{AD}(s)}{P_m(s)} \right)^{1-\gamma}, \qquad U_{mj}^{H} = \frac{1}{1-\gamma} \sum_{s} \mathbb{P}(s) \left(\frac{W_{mj}(s)}{P_m(s)} \right)^{1-\gamma}, \tag{26}$$

where the superscript AD denotes workers with access to AD securities and H denotes hand-to-mouth workers with no access (i.e. in financial autarky). The variable U_{mj}^{H} is exactly the expected utility in (1).

Since ability draws x_{mj} are independent of access status, each type's region-sector choice follows the same Fréchet-logit formula as in the baseline model, now applied to each

type-specific utility,

$$\lambda_{mj}^{F} = \frac{(U_{mj}^{F})^{\frac{\kappa}{1-\gamma}}}{\sum_{j',m' \in M_i} (U_{m'j'}^{F})^{\frac{\kappa}{1-\gamma}}}, \qquad F = AD, H. \tag{27}$$

Total employment in (m, j) is a mixture across types,

$$L_{mj} = L_{mj}^{AD} + L_{mj}^{H}, \qquad L_{mj}^{AD} = C_1 \mu_i L_i (\lambda_{mj}^{AD})^{1-\frac{1}{\kappa}}, \qquad L_{mj}^{H} = C_1 (1-\mu_i) L_i (\lambda_{mj}^{H})^{1-\frac{1}{\kappa}}. \tag{28}$$

When $\mu_i = 0$, (28) collapses to (4), while when $\mu_i = 1$ all workers are insured. In the deterministic equilibrium both types face the same real wage with no risk, so that $\lambda_{mj}^{AD*} = \lambda_{mj}^{H*} = \lambda_{mj}^{*}$, and the share of insured workers in every region-sector pair is μ_i.

Asset market equilibrium. Define $B_{mj}(s) \equiv [C_{mj}^{AD}(s) - W_{mj}(s)] L_{mj}^{AD}$ as the net receipts (positive) or payments (negative) from the security paying off in state s, and let $Y_i(s)$ and $X_i(s)$ denote, respectively, the value of gross output of, and expenditure on, tradable intermediate goods in country i in state s. Country i's net exports in state s are

$$NX_i(s) \;\equiv\; Y_i(s) - X_i(s). \tag{29}$$

With $B_i(s) \equiv \sum_{m \in M_i} \sum_j B_{mj}(s)$ denoting country i's total net holdings of the security, the country's state-s balance-of-payments condition becomes

$$NX_i(s) + B_i(s) = 0, \qquad \text{for all } s, \tag{30}$$

subject to the ex-ante present-value budget constraint

$$\sum_{s=1}^{S} Q_i(s)\, B_i(s) \;=\; 0. \tag{31}$$

Financial autarky imposes $B_{mj}(s) = 0$ for every s. Let $Y_i^{AD}(s) \equiv \sum_{m \in M_i} \sum_j L_{mj}^{AD} W_{mj}(s)$ denote the labor income of insured workers in country i. Since uninsured workers consume their wage in each state, insured workers as a group consume exactly their own labor income, pooled across states and, possibly, countries.

If the asset market is global, $\sum_{i=1}^{N} B_i(s) = 0$, and with world income as the numeraire in

each state, we have

$$\sum_{m,j} L_{mj}^{AD} C_{mj}^{AD}(s) = Y^{AD}(s) \equiv \sum_i Y_i^{AD}(s) \quad \Rightarrow \quad Q(s) = Y^{AD}(s)^{-\gamma} \left[\sum_{m,j} P_m(s)^{\frac{\gamma-1}{\gamma}} L_{mj}^{AD} \zeta_{mj} \right]^{\gamma}. \tag{32}$$

With full access, $\mu_i = 1$ for all i, $Y^{AD}(s) = 1$. Thus, the state-contingent price of AD securities is determined by a weighted average of final-good prices across regions, scaled by the aggregate resources of insured workers. Higher final-good prices increase the marginal utility of consumption, which raises the demand for AD securities and pushes up their price, given the supply of resources available to insured workers in each state.

If the asset market is specific to each country i, then $B_i(s) = 0$ for each i, and

$$\sum_{m \in M_i, j} L_{mj}^{AD} C_{mj}^{AD}(s) = Y_i^{AD}(s) \quad \Rightarrow \quad Q_i(s) = Y_i^{AD}(s)^{-\gamma} \left[\sum_{m \in M_i, j} P_m(s)^{\frac{\gamma-1}{\gamma}} L_{mj}^{AD} \zeta_{mj} \right]^{\gamma}, \tag{33}$$

so that the state-contingent price of AD securities is determined by a weighted average of final-good prices across regions within the country, as well as by the country-specific supply of resources to insured workers in state s, $Y_i^{AD}(s)$.

4.2 Volatility of Real Wages

From (25), the log real-income deviation of a worker with access to AD securities in region m, sector j, is $\hat{C}_{mj}^{AD}(s) - \hat{P}_m(s) = -(\hat{Q}_i(s) + \hat{P}_m(s))/\gamma$, so that their real-income volatility is

$$\Xi_{m,j}^{AD} \equiv \frac{1}{\gamma^2} \text{var}\big(\hat{Q}_i(s) + \hat{P}_m(s)\big) = \frac{1}{\gamma^2} \Big[\text{var}(\hat{Q}_i(s)) + 2\,\text{cov}(\hat{Q}_i(s), \hat{P}_m(s)) + \text{var}(\hat{P}_m(s)) \Big]. \tag{34}$$

The real-income volatility of a worker without access, $\Xi_{m,j}^{H} \equiv \text{var}(\hat{W}_{mj}(s) - \hat{P}_m(s))$, is the one in our baseline model.

In the presence of AD securities, sectoral wage risk can be fully insured by purchasing AD securities that facilitate transfers across states. However, region-specific price risk cannot be completely diversified away. This is because AD securities are cleared at the global or national level and therefore cannot fully offset subnational regional fluctuations in final-good prices. The regional heterogeneity in prices affects the marginal utility of transfers delivered through AD securities and, consequently, alters the effective per-unit value of AD securities differently across regions. Moreover, income risk depends not only on local price risk, $P_m(s)$, but also on its covariance with aggregate risk embedded in AD

securities, as captured by $Q_i(s)$.

In the special case of frictionless trade considered in Lemma 1, $P_m(s) = P_i(s)$ for all m, because all regions in the world face identical final-good price risks. Then (32) and (33) imply $\hat{Q}_i(s) = -\gamma \hat{Y}^{AD}(s) + (\gamma - 1)\hat{P}_i(s)$, where Y^{AD} is world (global market) or country i (national market) income of insured workers, so that $\hat{C}^{AD}_{mj}(s) - \hat{P}_m(s) = \hat{Y}^{AD}(s) - \hat{P}_i(s)$. We can then characterize the volatility of real wages as follows.

Lemma 2 (Volatility of real wages with partial AD access). *Under the assumptions of Lemma 1, the real-wage volatility of workers without access to AD securities in country i and sector j is $\Xi^H_{i,j} = \Xi_{i,j}$, given by* (14)*. The real-wage volatility of workers with access to AD securities is common to all sectors j in country i, and is given by:*
(1) if the AD security is global and $\mu_i = \mu$ for all i (including full access, $\mu = 1$),

$$\Xi^{AD}_{i,j} = \text{var}(\hat{P}_i(s)) = \sum_k \sum_{i'} \alpha_k^2 (\bar{\pi}_{i',k})^2 \sigma^2_{i',k}; \tag{35}$$

(2) if the AD security is country specific,

$$\begin{aligned} \Xi^{AD}_{i,j} = \text{var}(\hat{Y}^{AD}_i(s) - \hat{P}_i(s)) = \sum_k \Bigg[& \left(\frac{\theta_k}{1+\theta_k} s_{ik}(1 - \bar{\pi}_{i,k}) + \alpha_k \bar{\pi}_{i,k} \right)^2 \sigma^2_{i,k} \\ & + \left(s_{ik} \frac{\theta_k}{1+\theta_k} - \alpha_k \right)^2 \sum_{i' \neq i} (\bar{\pi}_{i',k})^2 \sigma^2_{i',k} \Bigg], \end{aligned} \tag{36}$$

where $s_{ik} \equiv \sum_{m \in M_i} \bar{W}_{mk} L_{mk} / \bar{Y}_i$ is the share of sector k in country i's income.

In the full insurance case with global AD securities, real-income volatility is determined by the variance of final-good prices, as all regions can fully insure against country-specific shocks. In contrast, when only country-level AD securities are available, real-income volatility of insured workers is equal to the volatility of country-level real income. In both cases, real-income volatility of insured workers is identical across subnational regions and sectors within a country, while uninsured workers bear the sector-specific real-wage risk in Lemma 1.[7]

[7] Since insured and uninsured workers share the same deterministic allocation, the sectoral composition of insured income coincides with that of total income up to the order of our approximation, so that s_{ik} is also the share of sector k in the income of insured workers. If the AD security is global but μ_i differs across countries, insured income $Y^{AD}(s)$ is no longer constant across states and $\Xi^{AD}_{i,j} = \text{var}(\hat{Y}^{AD}(s) - \hat{P}_i(s))$, with $\hat{Y}^{AD}(s) = \sum_{i'} \left(\mu_{i'} \bar{Y}_{i'} / \sum_n \mu_n \bar{Y}_n \right) \hat{Y}_{i'}(s)$.

4.3 Employment Reallocation

With full access to AD securities ($\mu_i = 1$ for all i), sectoral volatility has no impact on employment reallocation. First, as shown in Lemma 2, real income volatility is equalized across industries within a country. Second, while productivity shocks induce fluctuations in consumption around the average level, the expected consumption level remains equal to average wages because the productivity shock is of mean one. Consequently, variation in sectoral volatility does not alter either expected income or price volatility, and thus leaves employment reallocation unaffected. With partial access, uninsured workers still respond to sectoral volatility, and insured workers respond to the resulting changes in expected wages. The next proposition characterizes both responses.

Proposition 3 (Employment reallocation under partial AD access). *Under the assumptions of Proposition 1, consider an increase in sector j's own volatility $d\sigma_{i,j}^2 > 0$, holding all other variances fixed, and let*

$$\hat{\lambda}_{mj}^B \equiv -\frac{\gamma\kappa(1-\lambda_{i,j}^*)}{2(1-\xi_j)}\frac{\theta_j}{1+\theta_j}(1-\pi_{i,j}^*)\left[\frac{\theta_j}{1+\theta_j}(1-\pi_{i,j}^*)+2\alpha_j\pi_{i,j}^*\right]d\sigma_{i,j}^2 < 0 \tag{37}$$

denote the response in the baseline economy with financial autarky (Proposition 1), where $\xi_j \equiv -(\kappa-1)\frac{1-\theta_j\rho_j+\pi_{i,j}^*\theta_j(\rho_j+1)}{1+\theta_j} < 0$. Then:
(1) Workers without access to AD securities reallocate away from sector j,

$$\hat{\lambda}_{mj}^H = (1-\xi_j\mu_i)\,\hat{\lambda}_{mj}^B = -\frac{\gamma\kappa(1-\lambda_{i,j}^*)}{2(1-\tilde{\xi}_j)}\frac{\theta_j}{1+\theta_j}(1-\pi_{i,j}^*)\left[\frac{\theta_j}{1+\theta_j}(1-\pi_{i,j}^*)+2\alpha_j\pi_{i,j}^*\right]d\sigma_{i,j}^2 < 0, \tag{38}$$

where $\tilde{\xi}_j \equiv \frac{\xi_j(1-\mu_i)}{1-\xi_j\mu_i} \in [\xi_j, 0]$. All the results in Proposition 1 remain valid for $\hat{\lambda}_{mj}^H$, and the response is amplified by a larger share of insured workers,

$$\frac{\partial^2\hat{\lambda}_{mj}^H}{\partial\sigma_{i,j}^2\partial\mu_i} < 0.$$

(2) Workers with access to AD securities reallocate towards sector j,

$$\hat{\lambda}_{mj}^{AD} = \tilde{\xi}_j\,\hat{\lambda}_{mj}^H = \xi_j(1-\mu_i)\,\hat{\lambda}_{mj}^B \geq 0. \tag{39}$$

(3) Total employment in sector j, $\hat{\lambda}_{mj} \equiv \mu_i\hat{\lambda}_{mj}^{AD} + (1-\mu_i)\hat{\lambda}_{mj}^H$, falls in proportion to the share of uninsured workers,

$$\hat{\lambda}_{mj} = (1-\mu_i)\,\hat{\lambda}_{mj}^B \leq 0. \tag{40}$$

The baseline (Proposition 1) and full insurance are the two polar cases of Proposition 3: with $\mu_i = 0$, $\hat{\lambda}_{mj} = \hat{\lambda}^H_{mj} = \hat{\lambda}^B_{mj}$, while with $\mu_i = 1$, $\hat{\lambda}_{mj} = \hat{\lambda}^{AD}_{mj} = 0$. In this economy with partial insurance, workers without access to AD securities optimally reallocate away from sectors with higher productivity volatility. However, workers with access to AD securities internalize the wage gains generated by the exit of uninsured workers from these sectors: the resulting increase in average wages makes volatile sectors more attractive to insured workers. The sorting of insured workers into the more-volatile sector tames the wage increase in that sector. Therefore, a larger share of workers with access to AD securities amplifies the (negative) employment response of workers without AD securities, while dampening the response of aggregate sectoral employment.

4.4 The Gains from Trade

Let $\mathbb{U}_i^F \equiv \Gamma\left(\frac{\kappa+\gamma-1}{\kappa}\right)\left(\sum_{m\in M_i}\sum_j (U^F_{mj})^{\frac{\kappa}{1-\gamma}}\right)^{\frac{1-\gamma}{\kappa}}$ denote the average expected utility of workers of type $F = AD, H$ in country i, as in (18). Define country-wide average utility as the population-weighted mixture of the two types' average utility,

$$\mathbb{U}_i(\mu_i) \equiv \mu_i \mathbb{U}_i^{AD} + (1-\mu_i)\mathbb{U}_i^H, \tag{41}$$

and the gains from trade as $GT_i(\mu_i) \equiv \left(\mathbb{U}_i(\mu_i)/\mathbb{U}_i^A\right)^{\frac{1}{1-\gamma}}$. In autarky there is no trade deficit, $NX_i = B_i = 0$, nominal wages are constant across states, and real-income risk stems only from final-good prices, which are common across sectors. Hence, the autarkic equilibrium is the same for both types and for all μ_i: $\lambda^{AD,A}_{i,j} = \lambda^{H,A}_{i,j} = \lambda^A_{i,j} = \alpha_j$ and $\Xi^{AD,A}_{i,j} = \Xi^{H,A}_{i,j} = \Xi^A_{i,j}$, given by (15).

Proposition 4 (Gains from trade under partial AD access). *Under the assumptions of Proposition 2, the gains from trade are*

$$\begin{aligned} GT_i(\mu_i) \approx \prod_j \left(\bar{\pi}^j_{i,i}\right)^{-\frac{\alpha_j}{\theta_j}} \prod_j \left(\frac{\mu_i(\lambda^{AD}_{i,j})^{1-1/\kappa} + (1-\mu_i)(\lambda^H_{i,j})^{1-1/\kappa}}{(\lambda^A_{i,j})^{1-1/\kappa}}\right)^{\alpha_j\rho_j} \\ \times \left[\mu_i \prod_j \left(\frac{\lambda^{AD}_{i,j}}{\lambda^A_{i,j}}\right)^{-\frac{(1-\gamma)\alpha_j}{\kappa}} \prod_j \left(\frac{1-\frac{(1-\gamma)\gamma}{2}\Xi^{AD}_{i,j}}{1-\frac{(1-\gamma)\gamma}{2}\Xi^A_{i,j}}\right)^{\alpha_j}\right. \\ \left. + (1-\mu_i)\prod_j \left(\frac{\lambda^H_{i,j}}{\lambda^A_{i,j}}\right)^{-\frac{(1-\gamma)\alpha_j}{\kappa}} \prod_j \left(\frac{1-\frac{(1-\gamma)\gamma}{2}\Xi^H_{i,j}}{1-\frac{(1-\gamma)\gamma}{2}\Xi^A_{i,j}}\right)^{\alpha_j}\right]^{\frac{1}{1-\gamma}}, \end{aligned} \tag{42}$$

where $\lambda_{i,j}^A = \alpha_j$*, and* $\Xi_{i,j}^{AD}$*,* $\Xi_{i,j}^H$*, and* $\Xi_{i,j}^A$ *are given by Lemma 2 and* (15)*.*

The first two components of (42) affect insured and uninsured workers alike. In particular, employment changes among uninsured workers influence the welfare of insured workers through their impact on sectoral productivity generated by economies of scale. The final bracket represents the weighted average contribution of worker heterogeneity and uncertainty to welfare by the two groups. With $\mu_i = 0$, (42) collapses to the gains from trade in Proposition 2. With $\mu_i = 1$, we can compare the gains from trade under full insurance with those in our baseline economy.

Corollary 1 (**Gains from trade under full insurance**)**.** *Under the assumptions of Proposition 2, and holding* $\bar{\pi}_{ii}^j$ *fixed, the gains from trade with full access to AD securities relative to the baseline economy with financial autarky are*

$$\Delta GT_i \equiv \frac{GT_i(1)}{GT_i(0)} \approx \prod_j \left(\frac{1 - \frac{(1-\gamma)\gamma}{2} \Xi_{i,j}^{AD}}{1 - \frac{(1-\gamma)\gamma}{2} \Xi_{i,j}} \right)^{\frac{\alpha_j}{1-\gamma}} \prod_j \left(\frac{\lambda_{i,j}^{AD}}{\lambda_{i,j}} \right)^{\frac{\alpha_j}{\kappa}[(\kappa-1)\rho_j - 1]}, \tag{43}$$

where $\lambda_{i,j}$ *and* $\Xi_{i,j}$ *are the baseline employment shares and real-wage volatility, and* $\lambda_{i,j}^{AD}$ *solves* (17) *with* $\Xi_{i,j}^{AD}$ *instead of* $\Xi_{i,j}$*.*

Welfare unambiguously improves through the first term when $\Xi_{i,j}^{AD} < \Xi_{i,j}$, a condition that always holds in the symmetric world with global AD securities. With full insurance, uncertainty-induced employment reallocation across sectors is eliminated, so employment shifts are driven entirely by specialization, in contrast to the baseline case where they reflect both specialization and uncertainty. Therefore, the sign of the second term is determined by the interaction between economies of scale and uncertainty-driven employment reallocation. Specifically, strong economies of scale ($\rho_j > 1/(\kappa - 1)$) in sectors experiencing larger increases in uncertainty amplify the positive contribution of this channel to ΔGT_i for the shocked sector, whereas weak economies of scale attenuate it.

5 Empirical Evidence

In this section, we present suggestive evidence about our model predictions using data for community zones (CZ) across the United States. We first describe the data and the construction of the variables used for the analysis. Next, we document that uncertainty affects more negatively the employment growth of CZs with a larger share of employment in industries with strong economies of scale. Additionally, we document that more

openness of a CZ to international reinforces worker reallocation away from volatile sectors. We use some of the reduced-form evidence moments as targets for our calibration procedure in Section 6.

5.1 Data and Variables' Construction

We use employment data for the United States, by county and industry, from the County Business Patterns, for 1967-2017. We follow Autor et al. (2013) to clean the data and aggregate them into commuting zones (CZs). Data on U.S. sectoral employment and U.S. sectoral value of shipments are from the NBER Manufacturing Database, for 1969-2011. To be consistent with the sector classification in our quantitative analysis in Section 6, we aggregate the data into 2-digit ISIC industries for our baseline empirical results, and we report robustness analysis based on 3-digit SIC industries. We use as our variable of interest changes in employment at the CZ, or alternately, at the CZ-sector level, between 1990 and 2010. We remove CZ-sector pairs with zero employment either in 1990 or 2010. Our sample includes 721 CZs within the United States, and 17 manufacturing industries for which we can construct volatility measures as described below. See Appendix C for details.

To obtain a measure of uncertainty, we first compute growth in real value-added per worker by industry and year,

$$g_{j,t} = \log\left(\frac{VA_{j,t}}{Emp_{j,t}}\right) - \log\left(\frac{VA_{j,t-1}}{Emp_{j,t-1}}\right). \tag{44}$$

We then compute the change in uncertainty between 1970-1990 and 1990-2010 for industry j based on changes in the variance of real value-added per worker growth,

$$\Delta v_j = var(\underbrace{g_{j,t}}_{t=1990,\ldots,2010}) - var(\underbrace{g_{j,t}}_{t=1970,\ldots,1990}). \tag{45}$$

Next, we compute a region's change in uncertainty as a Bartik-type variable,

$$\Delta v_m \equiv \sum_j \frac{L_{mj,0}}{L_{m,0}} \Delta v_j, \tag{46}$$

where $L_{mj,0}$ is CZ m's employment in industry j, and $L_{m,0}$ is CZ m's total employment, in 1990.

We use as the measure of sectoral economies of scale the parameter $\tilde{\rho}_j \equiv \frac{\kappa-1}{\kappa}\rho_j\theta_j$ in (13), where we take both trade elasticity θ_j and the degree of economies of scale $\frac{\kappa-1}{\kappa}\rho_j$ from Bartelme, Costinot, Donaldson and Rodríguez-Clare (2025).[8] [9]

For some of our empirical evidence, we would need trade exposure at the CZ level. Because there are no available export and import data at the CZ geographic level, we measure each CZ's exposure to trade using its industry structure combined with the national openness of the industry. We first compute each industry's exposure to international trade by using the ratio of imports plus exports to total production. We then aggregate industries' exposure to trade in each CZ's using the CZ's industry structure as weights, following Autor et al. (2013) and Caliendo et al. (2019). In regressions at the CZ-industry level, we compute CZ-level trade exposure for each CZ-industry pair while excluding the pair's own employment from the calculation, in order to mitigate endogeneity concerns.

5.2 Results

We start by documenting how the correlation between sectoral employment and uncertainty is shaped by scale economies.

Figure 1a plots changes in log-employment between the 1990 and 2010, $\Delta \log L_m$, against changes in uncertainty, Δv_m, across CZs. The figure suggests that more uncertainty is associated to lower employment growth.

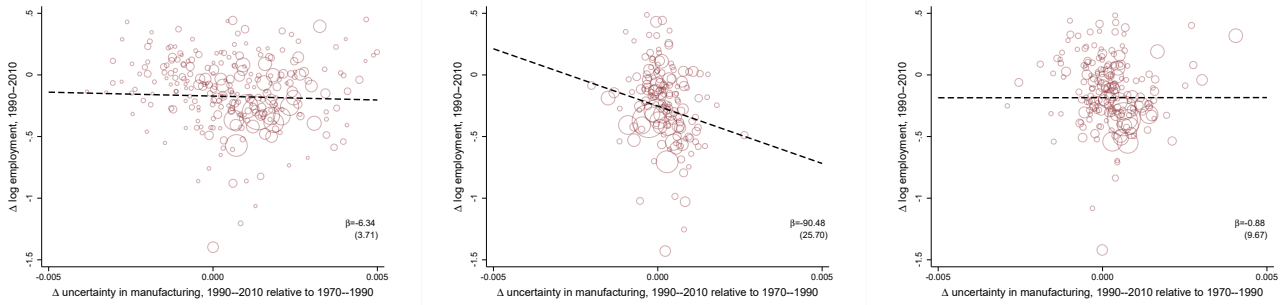
Are scale economies amplifying this negative effect? In Figures 1b and 1c, we divide industries into two groups based on whether their scale-economy parameters are above or below the median value. We then compute CZ-level changes in employment and uncertainty, for each group of industries. We find that industries with above-median economies of scale respond more to changes in uncertainty, as suggested by the theory.

[8] In Bartelme, Costinot, Donaldson and Rodríguez-Clare (2025), the degree of economies of scale is estimated using local sectoral employment. In our model, according to Assumption 1, we defined economies of scale ρ_j based on labor units. Given the relationship between labor units and employment in (4), the degree of economies of scale in terms of local sectoral employment in our model is $\frac{\kappa-1}{\kappa}\rho_j$, which corresponds to the estimate in Bartelme, Costinot, Donaldson and Rodríguez-Clare (2025).

[9] Their estimates span 16 2-digit ISIC sectors. We set the scale parameter for a more disaggregate sector when it belongs to more than one aggregate industry to an employment-based weighted average of $\tilde{\rho}_j$ across the aggregate industries. This assignment is particularly relevant for our robustness analysis that uses 3-digit SIC industries. We use the concordance between ISIC and SIC industries in https://www.freit.org/TradeResources/TradeConcordances.php.

Figure 1: Employment growth, economies of scale, and uncertainty.

(a) All industries (b) Strong economies of scale (c) Weak economies of scale



Note: The circles are proportional to the region employment level in 1990. Strong (weak) economies of scale refer to sectors with above (below) median values of the parameter $\tilde{\rho}_j$ defined in (13). The changes in employment and uncertainty are residualized with respect to average productivity growth in the corresponding sectors, capturing the first-order effects.

Formally, we estimate the following equation by Ordinary-Least-Squares (OLS),

$$\Delta \log L_m = \beta_0 + \beta_1 \Delta v_m + \beta_2 X_m + \epsilon_m, \tag{47}$$

where $\Delta \log L_m$ denotes changes in manufacturing employment for CZ m between 1990 and 2010, Δv_m is the change in volatility for CZ m computed as in (46), X_m denotes CZ-level controls, and ϵ_m is the error term.

Columns (1)-(3) of Table 1 corroborate the negative correlation documented in Figure 1: larger uncertainty is correlated with lower manufacturing employment growth, more so in industries with strong economies of scale, after controlling for the China shock, structural transformation, labor productivity growth, and demographic controls.

To use all the industries, rather than pooling them in two groups with high and low scale parameters, we use CZ-industry variables and estimate the following equation by OLS:

$$\Delta \log(L_{mj}) = \beta_0 + \beta_1 \Delta v_j + \beta_2 \Delta v_j \times \tilde{\rho}_j + \beta_3 \tilde{\rho}_j + \beta_4 X_{mj} + \epsilon_{mj}.$$

Here, $\Delta \log(L_{mj})$ denotes the change in industry j's employment for CZ m between 1990 and 2010, Δv_j denotes changes in volatility in industry j as defined in (45), and $\tilde{\rho}_j$ is the parameter capturing scale economies in industry j. The variable X_{mj} denotes CZ-industry level controls.

Columns (4) and (5) of Table 1 present the results. Conditional on sectoral uncertainty, employment growth at the industry-CZ level decreases with larger scale parameters. The standard deviation of the parameter $\tilde{\rho}$ across industries is 0.09, suggesting that that one

standard-deviation increase in the scale parameter would increase the magnitude of the (negative) response of employment growth to uncertainty by around 12 percent. It is worth noting that for all the values of $\tilde{\rho}$ for the industries in our sample, the effect of uncertainty on labor reallocation is always negative—that is, $\hat{\beta}_1 + \hat{\beta}_2\tilde{\rho} < 0$.

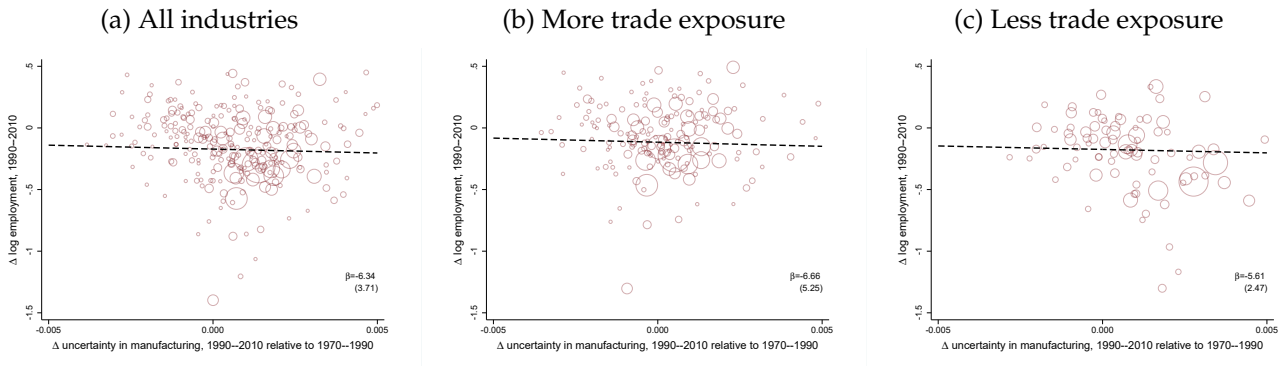
Table 1: Employment growth, scale economies, trade exposure, and uncertainty. OLS.

	$\Delta \log L_m$			$\Delta \log L_{mj}$		$\Delta \log \frac{L_{mj}}{L_j}$
	all industries	high $\tilde{\rho}_j$	low $\tilde{\rho}_j$	all industries		all industries
	(1)	(2)	(3)	(4)	(5)	(6)
Δv_m	-4.597*	-54.66**	-5.584			
	(2.539)	(26.54)	(8.840)			
Δv_j				-17.71***	57.93***	
				(1.837)	(17.50)	
$\tilde{\rho}_j$					-1.484***	
					(0.163)	
$\Delta v_j \times \tilde{\rho}_j$					-80.60***	
					(20.36)	
$\Delta v_j \times$ regional trade exposure						-98.20***
						(24.14)
Controls	yes	yes	yes	yes	yes	no
CZ fixed effects	no	no	no	yes	yes	yes
Industry fixed effects	no	no	no	no	no	yes
Obs	720	713	716	11,038	11,038	11,327
R-squared	0.110	0.211	0.112	0.421	0.518	0.424

Note: The regressions are weighted by initial manufacturing employment in the corresponding industry group. We examine changes in region-level employment in Columns (1)–(3), changes in region-industry-level employment in Columns (4) and (5), and changes in the region's share of employment within each industry in Columns (6). Changes in industry and CZ-level uncertainty are defined as in (45) and (46), respectively. In Columns (2)–(3), manufacturing industries are divided into two groups based on economies of scale: those with above-median (high) and below-median (low) $\tilde{\rho}_j$ as defined in (13). In Columns (1)–(3), the controls include region-level China shock and labor productivity growth (between 1990 and 2010), an employment-weighted average across industries, as well as the share of manufacturing employment. In Columns (4)–(5), the controls are extended to include a measure of industry-level China shock and labor productivity growth. Robust standard errors are in parentheses. * 10% ** 5% *** 1%.

Next, we present some evidence that supports the model's predictions that trade exposure may reinforce the response of employment growth to uncertainty. First, we show the results graphically in Figure 2. The pattern is consistent with the model's prediction: more openness amplifies the effect of uncertainty on employment growth at the CZ level. The effect, however, is not very sharp, most likely due to the fact that we had to impute the data on trade exposure for each CZ.

Figure 2: Employment growth, trade exposure, and uncertainty.



Note: The circles are proportional to the region employment level in 1990. More (less) trade exposure refers to regions with above (below) median values of the trade exposure. The changes in employment and uncertainty are residualized with respect to average productivity growth in the corresponding sectors, capturing the first-order effects.

Formally, we estimate the following equation by OLS:

$$\Delta \log \frac{L_{mj}}{L_j} = \beta \Delta v_j \times EX_m + CZ_m + I_j + \epsilon_{mj},$$

where $\Delta \log(L_{mj}/L_j)$ is the change in the share of industry j's employment in CZ m, and EX_m denotes the exposure to international trade of CZ m. CZ fixed effects are CZ_m and industry fixed effects are I_j.

Column (6) of Table 1 corroborates that more sectoral uncertainty tends to reallocate sectoral employment out from regions with larger trade exposure.

5.3 Robustness

We perform two robustness exercises.

First, we use an alternative measure for volatility from Alfaro, Bloom and Lin (2024) (ABL). Their measure is at the firm level, and reports the annual change in the volatility of the firm stock market returns. We calculate the firm-level average of annual changes between 1990–2020, assign firms to industries according to their main SIC code, and aggregate these averages across firms to get an industry-level measure of changes in volatility.

Columns (1)-(3) of Table 2 report results using ABL's uncertainty measure. Consistent with our previous results, we still find that uncertainty has negative effects on regional employment growth, and that the effect of uncertainty is larger for industries with stronger

Table 2: Robustness: Uncertainty Measure and Industry Aggregation. OLS

	Alternative uncertainty measure (ABL), 2-digit ISIC Industries,			Baseline uncertainty measure, 3-digit SIC Industries		
	$\Delta \log L_m$	$\Delta \log L_{mj}$	$\Delta \log \frac{L_{mj}}{L_j}$	$\Delta \log L_{mj}$	$\Delta \log L_{mj}$	$\Delta \log \frac{L_{mj}}{L_j}$
	(1)	(2)	(3)	(4)	(5)	(6)
Δv_m	-18.06**					
	(3.592)					
Δv_j		88.73***		-11.40***	59.89**	
		(22.85)		(1.050)	(26.82)	
$\tilde{\rho}_j$		1.157**			-4.770***	
		(0.522)			(0.262)	
$\Delta v_j \times \tilde{\rho}_j$		-112.81***			-76.43***	
		(24.91)			(29.24)	
$\Delta v_j \times$ regional trade exposure			28.08			4.831
			(19.11)			(21.05)
Controls	yes	yes	no	yes	yes	no
CZ FE	no	yes	yes	yes	yes	yes
IND FE	no	no	yes	no	no	yes
Obs	720	11,038	11,327	25,919	25,099	27,900
R-squared	0.181	0.538	0.422	0.210	0.293	0.208

Note: The regressions are weighted by initial manufacturing employment in the corresponding industry group. We examine changes in region-level employment in Columns (1), changes in region-industry-level employment in Columns (2) and (4)–(5), and changes in the region's share of employment within each industry in Columns (3) and (6). Changes in industry and CZ-level uncertainty are defined as in (45) and (46), respectively. In Columns (1), the controls include region-level China shock and labor productivity growth (between 1990 and 2010), an employment-weighted average across industries, as well as the share of manufacturing employment. In Columns (2), (4), and (5), the controls are extended to include a measure of industry-level China shock and labor productivity growth. Robust standard errors are in parentheses. * 10% ** 5% *** 1%.

economies of scale. However, with this uncertainty measure, the negative impact of uncertainty on local employment tends to decline with regional trade exposure, but the effect is not significant.

Our second robustness exercise uses more disaggregated industries, at 3 digit SIC rather than 2 digits ISIC. Our baseline results still hold. However, the trade-exposure measure has a positive effect but it is not significant.

Finally, Appendix D presents cross-country evidence on the effects of uncertainty and scale effects on employment growth. Appendix Table D.1 shows a negative correlation between changes in employment and changes in uncertainty, as well as a negative interaction between the scale parameter and changes in uncertainty. These results are consistent with our empirical evidence for the United States, though estimates are noisier due to a limited number of observations.

6 Calibration

In this section, we take the baseline model to the data. We first describe the data sources used, and then present the computation strategy. Finally, we discuss the calibration procedure.

We have rich sub-national data for the United States only. Hence, we focus on a version of the model where all countries except the United States have one region only.

Additionally, we assume that country-industry specific shocks $A_{i,j}(s)$ follow a log-normal distribution with covariance matrix $\mathbf{R}$, and the variance elements denoted by $\sigma^2_{i,j}$.

Finally, we extend the model to include sectoral input-output linkages as explained below.

Sectoral Input-Output Linkages. For our quantitative analysis, we extend the model to incorporate sectoral input-output linkages. We assume that production combines labor and goods from other sectors as inputs, as in (Caliendo and Parro 2015). Formally, the expenditure cost share of region m and sector j on inputs from sector k is δ^k_{mj}, while the labor share is δ^L_{mj}, with $\sum_k \delta^k_{mj} + \delta^L_{mj} = 1$. We define ζ^k_{mj} as the (j,k) element of the Leontief inverse matrix $[\mathbf{I} - \mathbf{\Phi}_m]^{-1}$, where $\mathbf{I}$ is an $J \times J$ identity matrix, and the (j,k) element of $J \times J$ matrix $\mathbf{\Phi}_m$ is δ^k_{mj}. The equilibrium of the model with sectoral input-output linkages is relegated to Appendix B. Mimicking Assumption 2, we impose that for all j the following condition is satisfied,

$$\sum_k (\tilde{\rho}_k - \frac{\theta_k}{\kappa}) \alpha_k \zeta^j_{mk} \theta_j < 1.$$

Bartelme, Jiang, Kucheryavyy and Rodríguez-Clare (2025) show that for $\kappa \to \infty$ and a small open economy, this condition is sufficient for existence, uniqueness, and an interior equilibrium, as well as for positive gains from trade. Our restriction is heuristic. However, by imposing this restriction in the calibration and simulation procedure described below, uniqueness is always achieved.

The gains from trade follow the expression in (19), with changes in utility per efficiency

unit in (20) adjusted to take into account sectoral linkages (see Appendix B.1),

$$\left(\frac{U_{mj}}{U_{mj}^{A}}\right)^{\frac{1}{1-\gamma}} \approx \underbrace{\left(\frac{\bar{\omega}_{mj}}{\bar{\omega}_{mj}^{A}}\right)}_{\text{Worker heterogeneity}} \times \underbrace{\prod_{k}\left(\frac{\bar{\pi}_{mm}^{k}}{\bar{\pi}_{mm}^{k,A}}\right)^{-\sum_{j}\frac{\alpha_{j}\zeta_{mj}^{k}}{\theta_{k}}}}_{\text{ACR}} \times \underbrace{\prod_{k}\left(\frac{\lambda_{mk}}{\lambda_{mk}^{A}}\right)^{\sum_{j}\alpha_{j}\zeta_{mj}^{k}\rho_{k}}}_{\text{Economies of scale}} \times \underbrace{\prod_{k}\left(\frac{\lambda_{mk}}{\lambda_{mk}^{A}}\right)^{-\frac{1}{\kappa}\sum_{j}\alpha_{j}\zeta_{mj}^{k}\rho_{k}}}_{\text{Interaction ES \& worker heterogeneity}}$$

$$\times \underbrace{\left(\frac{1-\frac{(1-\gamma)\gamma}{2}\Xi_{m,j}}{1-\frac{(1-\gamma)\gamma}{2}\Xi_{m,j}^{A}}\right)^{\frac{1}{1-\gamma}}}_{\text{Volatility of real wage}}, \tag{48}$$

with $\bar{\omega}_{mj} \equiv \bar{W}_{mj}/\prod_{k}\bar{W}_{mk}^{\sum_{j}\alpha_{j}\zeta_{mj}^{k}\delta_{mk}^{L}}$. Appendix B.2 and B.3 extend, respectively, Lemma 1 and Proposition 2 to the case of sectoral input-output linkages. Appendix B.4 to B.6 further extend these results to the case with AD securities.

6.1 Data

For our quantitative analysis, we need data on other countries in addition to the United States. Our main data sources are the OECD databases. To be as close as possible to the structure of the data in Section 5, we group industries into 20 aggregated sectors (see Appendix Table C.1), which include the 17 manufacturing sectors used in our empirical analysis, as well as agricultural, mining, and service sectors. Because computing the variance for country-sector-level productivity requires a relatively long time series data at the country-sector level, we only keep countries with at least 20 years of country-sector-level data on output and employment.[10] We are left with 26 countries, along with a constructed rest of the world. See Appendix C.1 for details.

Output, Value-added, and Employment. We obtain sectoral output, value-added, and employment for each country-sector in our sample from the OECD STAN Database.[11] The data span from 1970 to 2019 with shorter periods (at least 20 years) for some countries. Using this time series, we construct a measure of volatility of country-sector productivity

[10] An exception is China, which is not covered by the OECD databases on employment and value added. Given China's significant role in global trade, we calculate the variance of sectoral productivity growth for manufacturing sectors using data from the Annual Survey of Industry for the period 1998–2007; for other sectors, we use data from China's National Bureau of Statistics over the same period.

[11] For consistency, we also use OECD STAN for manufacturing sectors in the United States, rather than data from the NBER-CES Manufacturing Dataset, as in Section 5. Volatility of valued added per worker in manufacturing calculated from the OECD data is strongly correlated with the measure of volatility calculated using the NBER-CES Manufacturing Data (0.8). The OECD data also allow us add the agricultural, mining, and service sectors, in the United States.

for all the available years. Specifically, for each country and sector, we first deflate the value-added by CPI indicators from the Penn World Table (10.0) and then compute the standard deviation of yearly growth in value-added per worker, computed similarly to (44). As described below, we calibrate the volatility of country-sector-specific productivity in the model to match the standard deviation of real value-added per worker observed in the data.

For subnational data for the United States, we use, as in Section 5, output and employment aggregated into 721 the community zone (CZ) and 20 industries, from the U.S. County Business Patterns, for the year 2017.

International Trade Flows. We obtain sectoral trade flows between countries from the OECD Inter-Country Input-Output Tables, for 2017. Due to the lack of data on U.S. CZs' exports and imports, we make the following proportionality assumption. We assign U.S. sectoral exports to each country to each CZ using the share of each CZ's output in the U.S. total sectoral output. Similarly, we assign U.S. sectoral imports from each country to each CZ using the share of each CZ's expenditures in the U.S. total sectoral expenditures.

US Subnational Trade Flows. We use the Commodity Flow Survey (CFS) for 2017 to construct sectoral trade flows between CZs in the United States. One issue is that the trade flows in the survey are not recorded at the CZ geographic level. We adopt a similar procedure as in the literature (Allen and Arkolakis 2014, Monte, Redding and Rossi-Hansberg 2018, Fajgelbaum and Gaubert 2020) to assign trade flows between CZs. Specifically, we first parameterize the elasticity of trade costs to distance and then solve for the trade flows between CZs using the model structure. The details are provided in Appendix C.2.

6.2 Calibration Procedure

Solving for counterfactual equilibrium changes requires values for several sets of parameters: input-output linkages $\{\delta^k_{mj,i}, \delta^L_{mj,i}\}$; sectoral expenditure shares $\{\alpha_{i,j}\}$, which we allow to be country specific; the off-diagonal elements of the covariance matrix of country-sector productivity shocks, $\mathbf{R}$; the standard deviation of country-sector productivity shocks $\{\sigma_{i,j}\}$; trade elasticities $\{\theta_j\}$; economies of scale $\{\rho_j\}$; migration elasticity κ; and risk aversion coefficient γ. We calibrate these parameters as follows. Table 3 summarizes the parameter values and sources.

Externally Calibrated Parameters. We calibrate several parameters from the literature

Table 3: Parameter Values

Notation	Description	Value	Source/Moment
Panel A: Externally Calibrated Parameters			
κ	Migration elasticity	3	Fajgelbaum et al. (2019)-Monte et al. (2018)
$\{\theta_j\}$	Sectoral trade elasticities	5.66 (2.61)	Bartelme, Costinot, Donaldson and Rodríguez-Clare (2025)
$\{\rho_j\}$	Sectoral economies of scale	0.25 (0.16)	Bartelme, Costinot, Donaldson and Rodríguez-Clare (2025)
$\{\delta^k_{i,j}, \delta^L_{i,j}\}$	Sectoral input-output linkages	–	OECD Input-Output Tables (year 2017)
$\{\alpha_{i,j}\}$	Expenditure shares	–	OECD Input-Output Tables (year 2017)
$\mathbf{R}$	Cov. of productivity shocks	–	Cov. of growth of value-added per worker, OECD STAN Multi-factor model (Appendix E)
Panel B: Internally Calibrated Parameters (SMM)			
$\{\sigma_{i,j}\}$	S.D. of productivity shocks	–	S.D. of growth of value-added per worker, OECD STAN
γ	Risk aversion coefficient	4.2	OLS coefficient in Column (1) of Table 1

Note: For θ_j and ρ_j, we report the average and standard deviations across sectors. See Appendix Table C.1 for details.

and directly using the data. We set the migration elasticity to $\kappa = 3$, which is between the cross-state migration elasticity estimated in Fajgelbaum et al. (2019) and the cross-county elasticity of substitution in Monte et al. (2018). As in Section 5, we follow Bartelme, Costinot, Donaldson and Rodríguez-Clare (2025) and set the degree of scale of economies $\frac{\kappa-1}{\kappa}\rho_j$ and the sectoral trade elasticities θ_j to match their estimates (see Appendix Table C.1). With $\kappa = 3$, we recover ρ_j for each manufacturing sector j. For the agriculture, mining, and services sectors, we set $\rho_j = 0$ and $\theta_j = 4.5$ (from the aggregate elasticity estimates in Simonovska and Waugh 2013). We obtain the input-output parameters, for each country and sector, and sectoral expenditure shares, from the OECD Input-Output Tables for 2017.[12] We assume that for U.S. regions, the input-output parameters are equal to the national parameters, for each sector: $\delta^k_{mj,US} = \delta^k_{US,j}$ and $\delta^L_{mj,US} = \delta^L_{US,j}$, for all m. We follow a procedure based on a multi-factor structure with year-specific global, country, and sector components, and time series data on the growth of real value-added per worker, to calibrate the off-diagonal elements of the correlation matrix of productivity shocks across countries and sectors, $\mathbf{C}$ (see Appendix E for details). We calibrate the standard deviation of the shocks, $\mathbf{\Sigma} = \text{diag}(\sigma_{i,j})$, separately as we explain next, so that the covariance matrix of productivity shocks is $\mathbf{R} = \mathbf{\Sigma C \Sigma}$.

Internally Calibrated Parameters. We use a simulated method of moments to calibrate

[12]Expenditure shares $\{\alpha_{i,j}\}$ are computed as $\frac{\text{Output}_{i,j}+\text{M}_{i,j}-\text{E}_{i,j}-\text{INTER}_{i,j}}{\sum_j \text{Output}_{i,j}+\text{M}_{i,j}-\text{X}_{i,j}-\text{INTER}_{i,j}}$, where $\text{Output}_{i,j}$, $\text{M}_{i,j}$, $\text{X}_{i,j}$ and $\text{INTER}_{i,j}$ are output, imports, exports and intermediate-input use of goods from sector j, respectively, in each country i.

the volatility parameters $\{\sigma_{i,j}\}$ and risk aversion γ.

We calibrate the country-sector standard deviation of productivity shocks $\{\sigma_{i,j}\}$ to match the country-sector standard deviation of log value-added per worker observed in the data, for the period 1970-2019. To discipline the risk aversion coefficient, γ, we target the responses of employment to uncertainty across U.S. regions given by the OLS coefficient in Column (1) of Table 1. To such end, we need to separately match the U.S. sectoral standard deviation of value-added per worker in the periods 1970–1990 and 1990–2010, respectively, and compute the change in regional employment between 1990 and 2010, to perform the same regression as in (47).

We pair the average economy in the model, with data for 2017. The main restriction for this choice is the availability of intra-national trade flows for the United States, which are available for the year 2017. We describe the algorithm next.

Algorithm for SMM Procedure. We implement a simulated method of moments (SMM) procedure using hat-algebra techniques à la Dekle et al. (2008), adapted to incorporate uncertainty. The procedure takes as given: $\{\delta_{i,j}^k, \delta_{i,j}^L\}$, expenditure shares $\{\alpha_{i,j}\}$, economies of scale $\{\rho_j\}$, and trade elasticities $\{\theta_j\}$. we pair the observed economy in 2017 with the model's average economy. The calibration of volatilities $\{\sigma_{i,j}\}$ and risk aversion γ proceeds in two steps.[13]

1. Calibration of volatility standard deviation of productivity shocks, $\{\sigma_{i,j}\}$.

   1.1 For each (i,j), we draw 100 realizations of productivity shocks from a log-normal distribution with parameters $\mathcal{N}\left(-\frac{\sigma_{i,j}^2}{2}, \sigma_{i,j}^2\right)$, where the choice of the mean ensures that the productivity shocks have unit expectation.

   1.2 Conditional on observed employment shares in 2017, denoted by λ_{mj}^{17}, and our guessed $\{\sigma_{i,j}\}$, we compute equilibrium changes from the average to each simulated state, $s = 1, \cdots, 100$, using standard hat algebra methods (see Appendix B).[14]

   1.3 Using the 100 simulated equilibria, we compute the standard deviation of the real value added per worker growth. For the United States, we aggregate the

[13] Under the restrictions in parameters in Assumption 2, the model does not generate corner solutions in sectoral employment in any location. We match zero employment in the data by assuming that fundamental productivity is zero in those regions and sectors.

[14] Conditional on employment shares, in each state s, our model is a standard multi-sector Eaton-Kortum model with sectoral input-output linkages (e.g., Costinot and Rodríguez-Clare 2014, Caliendo and Parro 2015).

model-generated real value added per worker across regions using 2017 employment weights.

1.4 We iterate over $\{\sigma_{i,j}\}$ until the model-generated standard deviation of real value added per worker matches the corresponding data moment calculated for 1970-2019. We denote the calibrated volatilities as $\{\sigma_{i,j}^{70-19}\}$.

The first step of the algorithm does not use the risk aversion parameter γ. We proceed to calibrate this parameter using the ex-ante labor allocation responses to changes in volatility and targeting the OLS estimate of the coefficient β_1 in (47), reported in Column (1) of Table 1.

2. Calibration of the risk aversion parameter, γ.

2.1 Fix $\{\sigma_{i\neq US,j}^{70-19}\}$ from Step 1, and consider a counterfactual change in US volatility from $\sigma_{US,j}^{70-19}$ to $\sigma_{US,j}^{'}$, for each j.

a. For a given γ, we guess a new employment share $\lambda_{mj}^{'}$, draw 100 shocks using the counterfactual volatility matrix, $\{\sigma_{i\neq US,j}^{70-19}, \sigma_{US,j}^{'}\}$, and compute the equilibrium hat-changes between the average economy and each state s repeating the procedure in Step 1.

b. We iterate on $\lambda_{mj}^{'}$ until the guessed and implied employment shares coincide.

2.2 We repeat the procedure in 2.1: First, $\sigma_{US,j}^{'} = \sigma_{US,j}^{70-90}$ calibrated to match the observed volatility of real value added per worker growth across US region-sector pairs, in 1970-1990, and obtain employment share λ_{mj}^{90}; and second, $\sigma_{US,j}^{'} = \sigma_{US,j}^{90-10}$ calibrated to match the analogous moments in 1990–2010, and obtain employment shares λ_{mj}^{10}.

2.3 Using simulated employment allocations, we estimate (47) by OLS,

$$\Delta \log L_m = \beta_0 + \beta_1 \Delta v_m + \beta_d X_m + \epsilon_m.$$

We construct $\Delta \log L_m = \log L_m^{10} - \log L_m^{90}$, where $L_m^{10} = L_{US}^{10} \sum_j \lambda_{mj}^{10}$ and $L_m^{90} = L_{US}^{90} \sum_j \lambda_{mj}^{90}$, with L_{US}^{10} and L_{US}^{90} denoting total U.S. employment in 2010 and 1990, respectively. We use the model-generated employment weights for 1990 to construct the volatility measure Δv_m as in (46). We include as controls the model-generated (change in) log average real wage in industry j and region

m, the share of workers in manufacturing sectors in region m, and the share of trade from China.

2.4 We iterate on γ until the model-implied β_1 matches the empirical estimate reported in Column (1) of Table 1.

6.3 Calibration Results

Table 4 compares the predictions of the effect of changes in uncertainty on regional employment levels between the model and the data, for CZs in the United States. We find that with a risk aversion coefficient of 4.2, the model-generated data matches the observed response of CZ-level employment to uncertainty, as shown in Columns (1) and (2). Our estimate of relative risk aversion is within the range of estimates in the finance literature (see Elminejad, Havránek and Irsova 2022, for a meta-analysis). The shock correlation structure across countries and sectors is important for the calibrated value of γ. If we assumed zero shock correlation, i.e. $\mathbf{R}$ being diagonal, our algorithm would calibrate a risk aversion parameter of 7.2, much higher than baseline value of 4.2.

Columns (3)-(6) report the un-targeted responses of CZ-sector level employment to uncertainty and how this effect varies with sectoral economies of scale, and CZ trade exposure. Our model captures fairly well the pattern observed in the data, but the magnitude of these reduced-form effects are much smaller relative to the data.

7 Quantitative Analysis

Armed with the calibrated model, we start by documenting the sources of real income volatility in the calibrated economy. We then perform several counterfactual exercises to shed light on the role of uncertainty and scale economies in shaping specialization patterns and the gains from trade. Each exercise simulates the economy for 100 realizations of the matrix of shocks. We keep these realizations constant across different counterfactual exercises to ensure that the results are not driven by variation in these draws.

Table 4: Employment Growth and Uncertainty, by CZs and Industries.

	Targeted		Untargeted			
	$\Delta \log L_m$		$\Delta \log L_{mj}$		$\Delta \log \frac{L_{mj}}{L_j}$	
	(1) data	(2) model	(3) data	(4) model	(5) data	(6) model
Δv_m	-4.597*	-4.574***				
	(2.539)	(1.053)				
Δv_j			57.93***	8.265***		
			(17.50)	(1.538)		
$\tilde{\rho}_j$			-1.484***	0.056***		
			(0.163)	(0.004)		
$\Delta v_j \times \tilde{\rho}_j$			-80.60***	-15.30***		
			(20.36)	(1.568)		
$\Delta v_j \times$ regional trade exposure					-98.20***	-8.636***
					(24.14)	(1.516)
CZ FE	no	no	yes	yes	yes	yes
IND FE	no	no	no	no	yes	yes
Obs	720	720	11,038	11,439	11,327	11,438
R-squared	0.110	0.401	0.421	0.805	0.424	0.331

Note: The regressions are weighted by initial manufacturing employment in the corresponding industry group. We examine changes in region-level employment in Columns (1) and (2), changes in region-industry-level employment in Columns (3) and (4), and changes in the region's share of employment within each industry in Columns (5) and (6). Changes in industry and CZ-level uncertainty are defined as in equations (45) and (46), respectively. In Columns (1)–(2), the controls include region-level China shock and labor productivity growth, an employment-weighted average across the specified industries, as well as the share of manufacturing employment. In Columns (3)–(6), the controls are extended to include a measure of industry-level China shock and labor productivity growth. Robust standard errors are in parenthesis. * 10% ** 5% *** 1%.

7.1 Sources of Real Income Volatility

Lemma 1 shows that, with frictionless trade and i.i.d. shocks, real-wage volatility in sector j of country i has three sources: shocks to sector j in country i, which move both the wage and the price index; shocks to sector j in other countries, which affect the wage through foreign competition, partly offset by their effect on prices; and shocks to all other sectors, at home and abroad, which affect real wages only through the price index. We now quantify these sources in the calibrated model, which features input-output linkages, trade costs, and correlated shocks. In Appendix F, we linearize the equilibrium system of the full model and show that the variance-covariance matrix of region-sector real income is

$$\mathbf{\Omega}\,\mathbf{R}\,\mathbf{\Omega}^T, \tag{49}$$

where $\mathbf{\Omega}$ is the matrix of general-equilibrium elasticities of region-sector real income with respect to the productivity shocks $\mathbf{A}$, combining the responses of nominal wages and final-good prices, including through trade and input-output linkages, and $\mathbf{R}$ is the variance-covariance matrix of $\mathbf{A}$.

To understand the sources of real income volatility, we decompose $\mathbf{R}$ for each region-sector pair in the US as

$$\mathbf{R} = \underbrace{\mathbf{R}_{j,US}}_{34.5\%} + \underbrace{\mathbf{R}_{non-j,US}}_{20.0\%} + \underbrace{\mathbf{R}_{j,non-US}}_{39.0\%} + \underbrace{\mathbf{R}_{non-j,non-US}}_{23.4\%} + \underbrace{\mathbf{R}_{cov}}_{\text{-16.9\%}}, \tag{50}$$

where the five components capture, respectively, shocks originating from the same sector within the US, shocks from other sectors within the US, shocks from the same sector in other countries, shocks from other sectors in other countries, and the covariance among these shocks. We then plug these five components into equation (49) to compute their relative contributions to the overall variance.

On average across US region-sector pairs, shocks originating in the same sector account for most of real income volatility: 34.5 percent stems from shocks to the same sector within the US and 39 percent from shocks to the same sector in other countries. This finding suggests that sectoral competition and global linkages play an important role in shaping real income volatility.

These components map into the terms of Lemma 1. Shocks to the same sector within the US and in other countries correspond to the first and second terms in (14), respectively, while shocks to other sectors, which account for 20.0 and 23.4 percent of volatility, correspond to the third term. The large contribution of shocks to the same sector abroad reflects the foreign-competition channel in the second term. The covariance among shocks, which has no counterpart in Lemma 1, reduces volatility by 16.9 percent. The correlation structure of shocks also matters for the calibration: as discussed in Section 6.3, assuming uncorrelated shocks would require a coefficient of relative risk aversion of 7.2, rather than 4.2, to match the observed response of CZ employment to uncertainty.

Figure 3 presents the variance decomposition across sectors in the United States.

The left panel decomposes real income volatility by the origin of shocks across US sectors. Real income volatility is particularly pronounced in industries with large productivity shocks, such as petroleum, mining, and motor vehicles. This volatility is mostly accounted for by shocks to the same sector abroad, except in motor vehicles, where shocks to the same sector in the US dominate. The covariance term is negative in all sectors, and

volatility in services is close to zero. Across sectors, the correlation between overall real income volatility and productivity volatility is 0.57.

The right panel of Figure 3 decomposes real-income volatility into the contributions of nominal wages and final-good prices. Nominal wage volatility accounts for the majority of real-income volatility, whereas final-good prices contribute relatively little. Moreover, the covariance between nominal wages and final-good prices tends to dampen real-income volatility: because higher wages are passed through to local final-good prices, the induced price increases offset part of the nominal income gain, thereby muting fluctuations in real income.

The decomposition in Figure 3 clarifies the role of insurance in Section 7.4. By Lemma 2, workers with access to AD securities are insured against sector-specific risk: with global securities they bear only final-good price risk, and with country-level securities only aggregate country income risk. Because most real income volatility originates in sector-specific shocks, which largely offset each other at the country level, country-level AD securities remove most of the real income risk faced by workers. This explains why, in that case, uncertainty no longer interacts with economies of scale, and eliminating them has little effect on the gains from trade, as we show in Table 10.

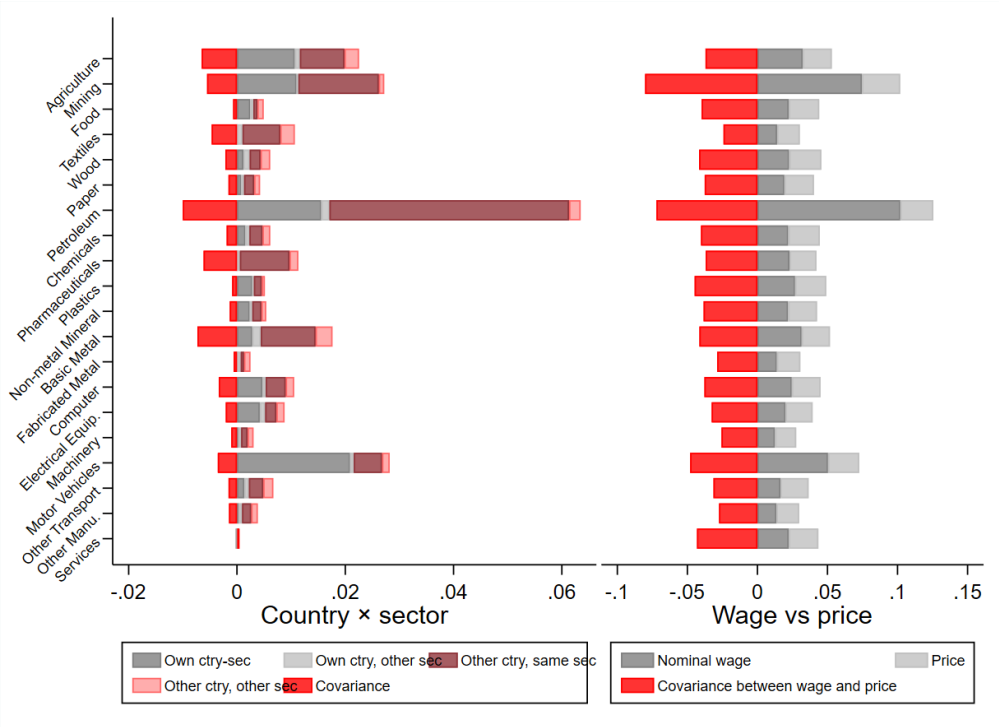
7.2 Economies of Scale, Uncertainty, and Regional Specialization

To understand the role of uncertainty in shaping workers' specialization patterns in the United States, we simulate the model with no uncertainty ($\sigma_{i,j} = 0$). In this case, uncertainty does not affect the ex-ante location and sectoral choices of workers. To analyze the interaction between uncertainty and scale economies, we repeat the exercise for an economy with no scale effects, $\rho_j = 0$ for all j. Results are shown in Figure 4.

The analytical case in Proposition 1 suggests that uncertainty in productivity should decrease employment growth, and the effect should be amplified with stronger scale economies. Quantitatively, the effect is relevant.

Panel (a) of Figure 4 shows that eliminating uncertainty tends to reallocate employment to regions in the middle of the United States (i.e. yellow areas). However, there is considerable heterogeneity across regions, ranging to losses of 0.33 percent for the 5th percentile of CZs and 0.12 percent for the median CZ, to gains of 0.14 percent for the mean and 1.46 percent for the 95th percentile CZ. Eliminating scale effects—and leaving only

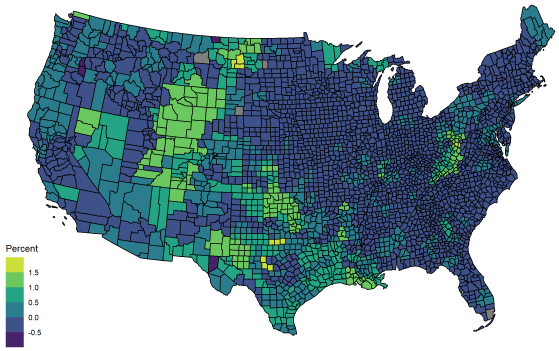
Figure 3: Decomposition of Calibrated Real Income Volatility.



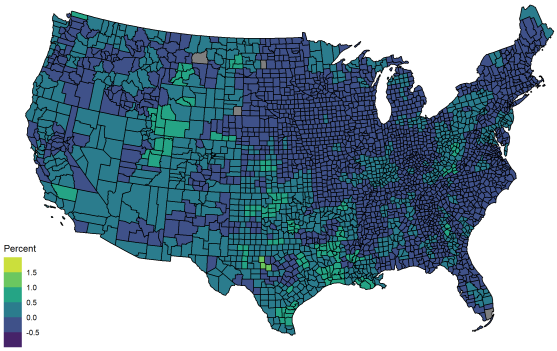
Note: In the left panel, we compute the variance of each component of real income volatility using equations (49) and (50). In the right panel, we decompose real-income volatility into the contributions of nominal wages and final-good prices, following the derivation in Appendix F. For each sector, we calculate the employment-weighted average across regions.

Figure 4: The Effects of Uncertainty on Employment Reallocation, By CZ.

(a) With Scale Effects



(b) Without Scale Effects



	mean	50th pc	5th pc	95th pc	minimum	maximum
With scale effects	0.14%	-0.12%	-0.33%	1.46%	-0.86%	2.82%
Without scale effects	0.06%	-0.05%	-0.21%	0.70%	-0.41%	1.47%

Note: Percentage change in CZ employment from the calibrated economy to an economy without uncertainty ($\sigma_{i,j} = 0$ for all i, j), with scale effects (calibrated ρ_j) in Panel (a), and without scale effects ($\rho_j = 0$ for all j) in Panel (b).

the forces of comparative advantages and geography in operation—mitigates the employment changes both for gains and losses, except for the Mountain region in the West where losses are exacerbated. Employment, however, would be reallocated away from the median CZ.

Who are the CZs gaining and losing according to our model predictions? Table 5 further lists the ten CZs with the largest employment losses and employment gains, respectively. Localities with the largest losses and gains are located in the South and Central regions –none of them is in the Northeast, Midwest, or California. Texas is the state with CZ's with both largest gains and losses.

To better visualize how uncertainty shapes local employment levels under scale effects and different degrees of trade openness, Panel (a) of Figure 5a plots the impact of eliminating uncertainty on employment against the variance of real wages in the calibrated economy, across CZs. Regions with a larger initial volatility would attract more workers when uncertainty is eliminated. This result is intuitive: risk-averse workers move away from regions and sectors with larger uncertainty, which lowers their utility. In line with the results in Figure 4, a world without economies of scale would be flatter, and the benefits of eliminating uncertainty are lower and similar across CZs.

Table 5: Smallest and Largest Employment Responses, Baseline to No Uncertainty.

Largest Employment Losses	With ES	Without ES	Largest Employment Gains	With ES	Without ES
Condon (OR)	-0.86	-0.22	Houma (LA)	1.86	0.94
Van Horn (TX)	-0.86	-0.23	Big Spring (TX)	1.93	0.79
Scobey (MT)	-0.66	-0.17	Worland (WY)	1.94	0.84
Clayton (NM)	-0.60	-0.14	Bluefield (WV)	1.96	1.01
Blytheville (AR)	-0.56	-0.41	Ardmore (OK)	1.96	1.22
Seymour (TX)	-0.52	-0.12	Elk City (OK)	2.03	0.73
Rosebud (SD)	-0.48	-0.39	Dickinson (ND)	2.22	0.95
Andrews (NC)	-0.45	-0.32	Buckhannon (WV)	2.31	1.19
Lakeview (OR)	-0.45	-0.22	Graham (TX)	2.55	1.18
Matador (TX)	-0.45	-0.38	Brady (TX)	2.82	1.47

Note: Ten CZs with the largest employment losses (gains) of moving from the calibrated economy to one with no uncertainty ($\sigma_{i,j} = 0$), with and without economies of scale (ES), respectively. We list the largest location in the CZ in terms of population (data from USDA).

Panel (b) of Figure 5a shows that removing uncertainty would reallocate employment towards CZ more exposed to trade, likely because these regions are specialized in industries with strong scale economies — in the absence of scale economies labor allocation does not change much for these regions.

7.3 The U.S. Gains from Trade

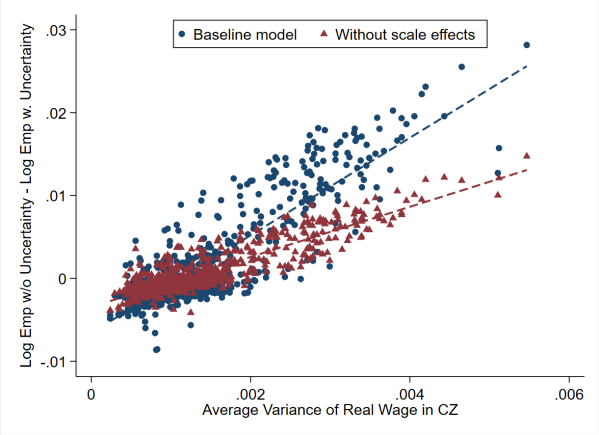
We now apply the calibrated model to understand the forces shaping the U.S. gains from trade—comparative advantage, economies of scale, uncertainty, and workers' heterogeneity. The gains from trade are defined in (19) as the change in the average expected utility from autarky to the calibrated economy, and we use the expression in (48). We set international trade costs for each CZ to infinity and calculate equilibrium variables under autarky. We compare the autarky and the baseline equilibrium, and repeat the exercise for an economy with no uncertainty, $\sigma_{i,j} = 0$, for all i, j, no scale economies, $\rho_j = 0$ for all j, and lower worker heterogeneity, $\kappa = 5$.

Table 6 summarizes the results. In the first row, we find that the overall gains from trade in our baseline model are 1.87 percent.[15] Removing aggregate uncertainty reduces the gains from trade from 2.27 to 1.87 percent. In Figure 6, we decompose this reduction into

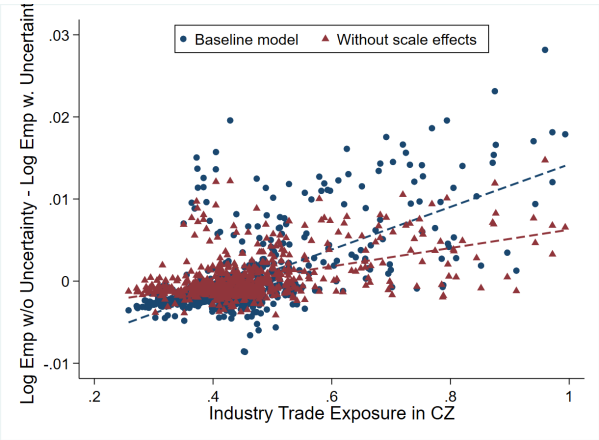
[15]Using the exact expression for expected utility rather than the second order approximation in (48) would only change the gains from trade by less than 0.01 percent.

Figure 5: Employment Reallocation, Uncertainty, and Scale Effects.

(a) Reallocation and Real-wage Variance



(b) Reallocation and Trade Exposure



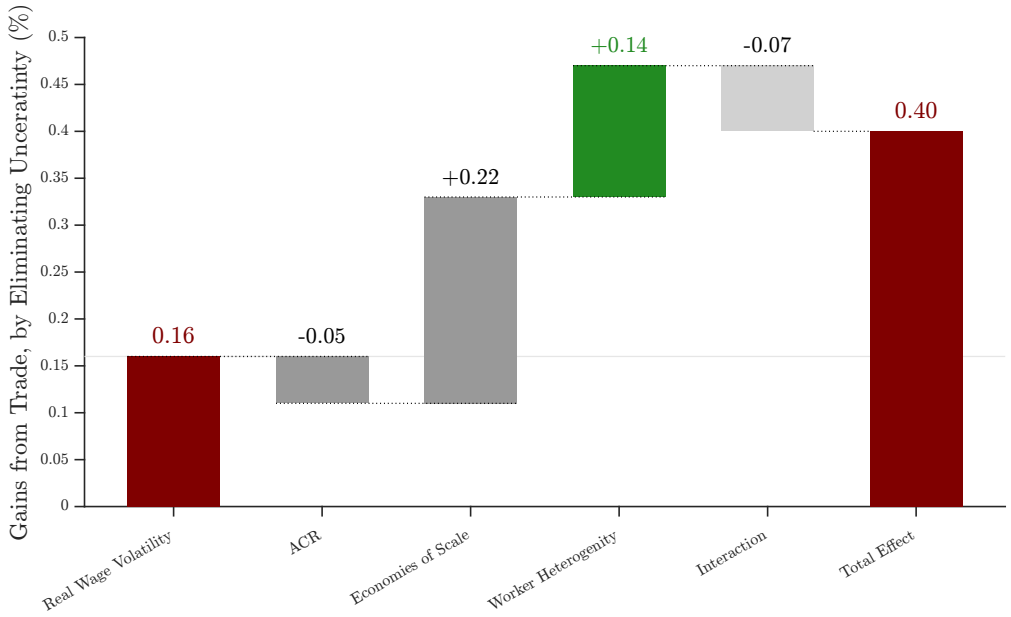
Employment changes from eliminating uncertainty against real-wage volatility in the calibrated equilibrium. For each CZ, real wage volatility is aggregated across industries using employment shares in 1990, while trade exposure is calculated as the ratio of exports plus imports over total output, as in Section 5.

five components based on (48). We find that the majority of the reduction is driven by two factors: the decline in real wage volatility, which is a direct consequence of eliminating uncertainty, and the interaction between economies of scale and uncertainty, which generates an additional reduction through their interaction effects.

This result suggests that larger wage volatility after trade openness dominates the role of trade in diversifying the sources of expenditures—and hence, the ability of trade of reducing the volatility of aggregate prices. In fact, the average volatility of log real wages across region-sector pairs across the United States increases from a variance of 0.0003 in autarky to 0.014 in our baseline economy, with some pairs reaching 0.08. As expected, when we aggregate at the CZ level, the variance in log real-wage decreases to 0.0013 in the baseline, with some regions reaching 0.005. Appendix Figure G.1 plots histograms to show the distribution of volatility changes across region-sector pairs and regions, respectively.

The decomposition in Section 7.1 helps explain why trade raises real income volatility. In autarky, US real wages respond only to domestic shocks through final-good prices. In the open economy, by contrast, shocks originating abroad account for 62.4 percent of real income volatility – 39.0 percent from shocks to the same sector in other countries, and 23.4 percent from shocks to other sectors abroad. Hence, the increase in volatility with openness should mostly reflects exposure to foreign shocks in the same sector, which reach US wages through competition in that sector's market.

Figure 6: Decomposition of the Gains from Trade: No Uncertainty versus Uncertainty.



Note: Each bar represents a term in equation (48).

In line with Proposition 1, the larger uncertainty in real wages drives workers away from sectors with large economies of scale, limiting their role. In fact, while uncertainty reduces the gains from trade by around 10-20 percent, with scale economies (2.27 vs 1.87 percent) or without them (2.27 vs 1.96 percent), there is little increase in the gains from trade if scale economies were removed in a deterministic environment, but much more in a risky environment (from 1.87 to 1.96 percent). That is, the interaction between uncertainty and scale economies brings down the benefits of specialization, as captured by the interaction term in Figure 6.

The third column of Table 6 shows results with a more elastic labor supply (i.e. $\kappa = 5$). When labor becomes more substitutable across locations and sectors, meaning that comparative advantage forces on the worker side are weaker, the gains from trade are lower, either in the presence or absence of scale economies. However, as highlighted by Proposition 2, differences between the economy with and without scale economies when $\kappa = 5$ tend be larger than the differences for the economy with $\kappa = 3$ since these agglomeration forces tend to increase with a more elastic labor supply, as captured by the third and fourth terms in (48).

Finally, the last column of Table 6 shows that if we shut down correlation of shocks across sector-region pairs in the United States, and countries, the gains from trade would

decrease considerably, from 1.87 to 1.69 percent — but less so if we also shut down economies of scale. This result points to the importance of considering shock correlation when modeling uncertainty, and is consistent with the decomposition in Section 7.1, whereby the covariance among shocks lowers real income volatility by 16.9 percent. [16]

How different would the magnitudes in Table 6 be if we applied directly the sufficient-statistic-type formula derived under frictionless trade and i.i.d. shocks in Proposition 2 (i.e. the version with input-output links in Appendix B)? In this case, we can use directly the data on trade and labor shares together with the calibrated elasticities. Gains from trade would be 1.70 percent, almost the same as in our exercise with i.i.d. shocks, while the ACR formula would implied gains of 2.2 percent, and adding to it scale effects would create negative gains (-1.5 percent). Further adding workers heterogeneity would reverse the negative effect, bringing gains to 1.8 percent. Uncertainty would only decrease gains by 0.08 percent.

Table 6: US Gains from Trade: Scale Effects, Uncertainty, and Worker Heterogeneity.

	Uncertainty	No uncertainty	$\kappa = 5$	i.i.d. shocks
Scale economies	1.87%	2.27%	1.81%	1.69%
No scale economies	1.96%	2.27%	1.93%	1.82%

Note: Gains from trade are calculated by comparing the autarky to the calibrated economy, using (19). For no uncertainty, we set $\sigma_{i,j} = 0$, for all i, j.For no scale economies, we set $\rho_j = 0$ for all j. For the baseline and no uncertainty scenario, we set $\kappa = 3$. For i.i.d. shocks, we set $\Sigma = 0$ (i.e. covariances across sectors and countries are zero).

The aggregate numbers in Table 6 mask substantial heterogeneity across regions. In Figure 8, we show regional gains from trade: We aggregate over sectors the expression in (48) using region-specific sector employment shares, $\sum_j (\lambda_{mj} / \sum_{j'} \lambda_{mj'}) \left(U_{mj}/U_{mj}^A\right)^{\frac{1}{1-\gamma}}$. Table 7 shows summary statistics across CZs.

The mean and median CZ gains from trade in our baseline calibration, are 1.74 and 1.77 percent. However, some CZs gain little, as reflected by the low minimum gain. Eliminating uncertainty would produce more than 20 percent higher gains for the average

[16] If one were to applied the sufficient-statistic-type formula in Proposition 2 (i.e. the version with input-output links in Appendix B) derived under frictionless trade and i.i.d. shocks, using directly the data and our calibrated parameters, gains from trade would be 1.70 percent, almost the same as in our exercise with i.i.d. shocks. With only the forces created by comparative advantage gains would increase to 2.2 percent; adding scale effects would create losses (-1.5 percent), while further adding workers heterogeneity would reverse the negative effect, bringing gains to 1.8 percent. Uncertainty only decreases gains by 0.08 percent.

Table 7: Regional Gains from Trade. Summary Statistics.

	mean	50-pc	5-pc	95-pc	minimum	maximum
$\kappa = 3$						
Baseline	1.74%	1.77%	1.20%	2.19%	0.50%	4.45%
No uncertainty	2.17%	2.16%	1.77%	2.60%	0.74%	4.85%
No scale economies	2.04%	2.05%	1.42%	2.60%	-0.75%	3.85%
No scale economies & No uncertainty	2.38%	2.39%	1.70%	2.99%	-0.58%	4.07%
$\kappa = 5$						
Baseline	1.65%	1.67%	1.11%	2.04%	0.55%	5.04%
No uncertainty	2.10%	2.07%	1.77%	2.46%	1.30%	5.49%
No scale economies	2.00%	1.99%	1.54%	2.45%	-0.05%	3.39%
No scale economies & No uncertainty	2.36%	2.37%	1.81%	2.86%	0.15%	3.62%

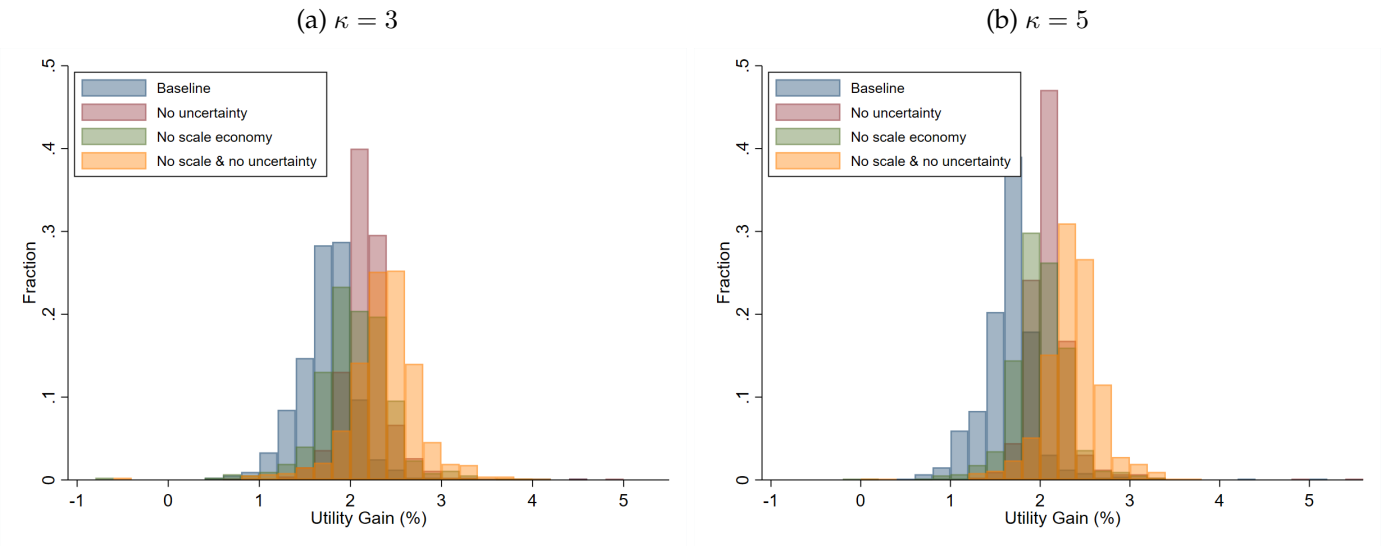
Note: Gains from trade in each CZ, $\sum_j (\lambda_{mj} / \sum_k \lambda_{mk}) \hat{U}_{mj}^{\frac{1}{1-\gamma}}$. For the model without uncertainty, we set $\sigma_{i,j} = 0$, for all i, j. For the model with no scale effects, we set $\rho_j = 0$, for all j. Statistics calculated over values across CZ's.

CZ. Simulating the model with both no uncertainty and no economies of scale reveals that economies of scale are also important in delivering more positive gains from trade for some regions; the model without those two forces would deliver average gains 37 percent higher than the baseline, but some locations would experience loses from trade. Finally, more responsive labor (i.e. $\kappa = 5$) produces lower gains from trade, and in line with the results in Table 6 differences between the economy with scale economies and the one without tend to increase with more responsive labor.

Figure 7 shows the distribution of gains across the United States on the form of histograms. It is clear from the figures that not only the mean is lower for the baseline economy, but also the whole distribution is a shift to the left relative to all the other cases, followed by the case with no scale economies, no uncertainty, and both no uncertainty and no scale economies—preserving the ranking observed in the mean values in Table 7.

Figure 8 shows the spatial distribution of gains from trade in the US map. Workers in the Bay area, and the Northeast corridor tend to enjoy larger increases in utility from trade openness. In contrast, workers in the Mountain region, parts of Texas, and parts of the Rust Belt (darker blue) would gain little from trade openness. The role of scale economies can be seen inspecting this figure: scale economies are strong enough that areas in northern Georgia, for instance, would lose from opening up to trade if we eliminated those

Figure 7: Regional Gains from Trade, distribution.



Note: Gains from trade in each CZ, $\sum_j (\lambda_{mj} / \sum_k \lambda_{mk}) \hat{U}_{mj}^{\frac{1}{1-\gamma}}$. For the model without uncertainty, we set $\sigma_{i,j} = 0$, for all i, j. For the model with no scale effects, we set $\rho_j = 0$, for all j.

forces, even if uncertainty were minimal.

Table 8 lists the communities with largest gains and the largest losses, both in the baseline model and the model with no uncertainty. Uncertainty decreases the gains from trade for the top winners between 0.3 and 0.4 percentage points. Focusing on El Paso (TX) as an example reveals that while the ACR model predicts a positive gain from trade of 2.53 percent, our baseline model predicts a much smaller overall gain of 0.81 percent. This reversal is driven by substantial losses from economies of scale (-1.35 percent) and increased uncertainty (-0.35 percent), both computed conditional on labor allocations, in the baseline equilibrium and under autarky. In line with the scale-related losses, the region also experiences an employment decline of 3.95 percent when moving from autarky to trade openness. Moreover, when labor mobility is more elastic (i.e. κ= 5), the trade gains lower to 0.77 percent, driven by stronger employment responses (-6.42 percent).

Conversely, when either economies of scale or uncertainty is removed, the gains from trade for El Paso turn much larger—1.38 and 1.50 percent, respectively. Overall, an interesting pattern emerges: While the top-ten winners remain about the same in the baseline and in a world with no uncertainty, the communities with the smallest gains in the baseline change considerably—many of them would not be the ones with the smallest gains if uncertainty were absent.

Table 8: Smallest and Largest Responses in Regional Gains from Trade.

Baseline				No Uncertainty			
Smallest		Largest		Smallest		Largest	
Andrews (NC)	0.50	Devils Lake (ND)	2.56	Andrews (NC)	0.74	Las Vegas (NV)	2.97
Rome (GA)	0.55	Alamosa (CO)	2.56	Rome (GA)	0.86	Alamosa (CO)	2.99
Bellingham (WA)	0.61	Miami (FL)	2.61	Kosciusko (MS)	1.29	San Francisco (CA)	3.00
El Dorado (AR)	0.63	Lake Providence (LA)	2.62	Matador (TX)	1.31	Lake Providence (LA)	3.03
Bonners Ferry (ID)	0.68	Limon (CO)	2.80	Bonners Ferry (ID)	1.38	Limon (CO)	3.22
Many (LA)	0.75	John Day (OR)	2.87	Many (LA)	1.44	John Day (OR)	3.26
El Paso (TX)	0.81	Burns (OR)	3.38	El Paso (TX)	1.50	Burns (OR)	3.67
Middlesborough (KY)	0.87	Murdo (SD)	3.52	Van Horn (TX)	1.50	Murdo (SD)	3.83
Brady (TX)	0.91	Arlington (VA)	4.00	Condon (OR)	1.50	Arlington (VA)	4.44
Oklahoma City (OK)	0.97	New York (NY)	4.45	Seymour (TX)	1.51	New York (NY)	4.85

Note: Ten CZs with the largest or smallest gains from trade, with and without uncertainty, respectively. For each CZ, we list the largest location in the CZ in terms of population (data from USDA).

Further examining Figure 9, which shows labor reallocation across regions from autarky to trade openness, suggests that the regions that gain less from openness to trade are the ones driving employment away. In the baseline model, the change in employment shares is -0.63 percent for the average CZ. Economies of scale, together with $\kappa = 5$, deliver the strongest employment responses with the average CZ having a 1.36 percent decrease in its employment share relative to autarky — doubling the magnitude delivered by our baseline model with a lower migration response (i.e. $\kappa = 3$). In the model with no scale economies, moving from autarky to the trade equilibrium slightly increases the average employment share (0.09 percent), suggesting that the effect of economies of scale changes the sign of the effect of trade openness on employment. If uncertainty were removed, the average change in employment shares would be similar to the one in the baseline model. The interaction between uncertainty and economies of scale further shifts the effect of trade openness on employment shares: In the model without those two forces, the employment share of the average CZ would increase by 0.19 percent when the economy moves from the autarky.

Finally, Figure 10 shows the correlation between the gains from trade (employment reallocation) across CZ for the model without scale effects and uncertainty (only comparative advantage forces and workers heterogeneity) and the baseline model, the model without scale effects, and the model without uncertainty, respectively. There is a positive correlation across all models, meaning that CZs with predicted higher gains and higher employment reallocation in one model are also the ones with predicted higher magni-

tudes in the remaining setups. However, there is large heterogeneity in the magnitude across localities, and consistent with previous results, some CZs, which experience gains in the baseline, would experience losses if scale effects were not operating. In contrast, on the upper end of gains, the absence of scale effects would increase gains, while the presence of uncertainty would decrease them—more so, in the baseline economy where both uncertainty and economies of scale are presence. A similar pattern is observed for employment reallocation across locations.

Overall, these results reinforce the idea that in the presence of uncertainty, scale effects not always increase the gains from trade. Our calibration suggests that these agglomeration forces have in fact a negative effect on the U.S. aggregate gains from trade. In a final result, we capture the interaction between uncertainty and economies of scale in shaping the gains from trade and employment reallocation across regions and sectors in the United States by a regression similar to the ones in Table 4. In Table 9, we show the results of estimating by OLS the effect of the change in real-wage volatility between autarky and the calibrated economy on changes in utility and employment, across CZ-sector pairs. We further interact the change in volatility with the parameter capturing the strength of scale economies in a sector. We find that larger increases in the real-wage variance from autarky to the baseline economy are associated with lower gains from trade and less employment, with effects that are amplified by economies of scale.

Table 9: Interaction between Uncertainty and Economies of Scale. OLS.

	$(U_{mj}/U_{mj}^A)^{\frac{1}{1-\gamma}}$	L_{mj}/L_{mj}^A
$\Xi_{mj} - \Xi_{mj}^A$	-1.58***	-3.18***
	(0.076)	(0.241)
$(\Xi_{mj} - \Xi_{mj}^A) \times \rho_j$	-5.37***	-15.18***
	(0.423)	(1.350)
CZ FE	yes	yes
IND FE	yes	yes
Obs	13,579	13,579
R-squared	0.858	0.841

Note: Changes in utility (per efficiency unit) and employment, from autarky to baseline economy, for each CZ-sector pair in the United States. Robust standard errors are in parentheses. * 10% ** 5% *** 1%.

7.4 Gains from Trade with AD Securities

We now examine how insurance alters our baseline results, holding all calibrated parameters fixed. We first introduce global AD markets, where workers worldwide can trade state-contingent claims. As shown in Panel (A) of Table 10, eliminating uncertainty still increases the gains from trade, from 1.88 to 2.27 percent, similar to our baseline result. However, the underlying source of trade-induced real income volatility changes. Because AD securities allow workers to fully insure against sector-specific shocks, real income volatility no longer varies across sectors within a region, eliminating the interaction between uncertainty and economies of scale. Instead, volatility arises from exposure to global AD prices, which are determined by global shocks reflected in final-good prices across countries, as described by equations (32) and (34).

We next consider the case in which AD markets are segmented by country, so that workers within each country trade state-contingent claims only among themselves. In this case, eliminating uncertainty has little effect on the gains from trade. This result follows from Lemma 2, which shows that, under certain assumptions, real income volatility is determined by aggregate country-level income volatility. The latter tends to be small because sectoral fluctuations are offsetting at the aggregate level. In particular, Caselli et al. (2020) find that changes in trade barriers from 1972 to 2007 have nearly null effects on aggregate income volatility in the United States.

These results are consistent with the sources of real income volatility documented in Section 7.1. With access to AD securities, insured workers in a country face the same volatility in all sectors. Because most real income volatility originates in shocks to the same sector, at home and abroad, and these shocks largely offset each other once income is pooled across sectors, little risk remains, and uncertainty barely affects the gains from trade.

Finally, we consider the more realistic case in which only a fraction of workers have access to AD securities, while keeping AD markets at the country level. We calibrate two benchmark values for μ_i: (1) $\mu_i = 0.7$, implying that 30 percent of workers are hand-to-mouth consumers, based on Kaplan, Violante and Weidner (2014); and (2) $\mu_i = 0.3$, reflecting that only roughly one-third of unemployed workers claim unemployment insurance (many unemployed workers are ineligible), as documented by Auray, Fuller and Lkhagvasuren (2019). Panel (A) of Table 10 reports the results. Two findings emerge. First, uninsured workers obtain smaller gains from trade than insured workers because they bear greater real-income risk. Second, increasing the share of uninsured workers strengthens the neg-

Table 10: Gains from Trade under Alternative Model Specifications

	With Uncertainty		Without Uncertainty	
	Scale Economies	No Scale Economies	Scale Economies	No Scale Economies
Baseline model	1.87%	1.96%	2.27%	2.27%
(A) With AD securities				
Global—fully insured	1.88%	1.88%	2.27%	2.26%
National—fully insured	2.28%	2.26%	2.26%	2.26%
National—70% insured				
insured workers	2.19%	2.21%	2.26%	2.25%
uninsured workers	2.03%	2.06%	2.26%	2.25%
National—30% insured				
insured workers	2.10%	2.16%	2.26%	2.26%
uninsured workers	1.95%	2.01%	2.26%	2.26%
(B) Alternate trade & ES values	3.74%	3.84%	4.53%	4.15%
(C) Nested migration choices	1.85%	1.95%	2.28%	2.27%
(D) Region-sector shocks	1.83%	1.93%	2.23%	2.27%

Notes: This table reports gains from trade under alternative combinations of uncertainty, scale economies, and insurance arrangements.

ative interaction between uncertainty and economies of scale. Proposition 4 shows that economies of scale operate through a common channel for both groups of workers; therefore, the amplified interaction effect also reduces the welfare gains of insured workers.

7.5 Sensitivity Analysis

Alternative Values of Trade and Scale Elasticities. As a robustness exercise, we replace the estimates from Bartelme, Costinot, Donaldson and Rodríguez-Clare (2025) with the sectoral trade and scale elasticities estimated by Lashkaripour and Lugovskyy (2023). Because Lashkaripour and Lugovskyy (2023) report lower trade elasticities across sectors (3.7 on average versus 5.7 in our baseline calibration), the implied baseline gains from trade are larger, rising from 1.87% to 3.74%, as shown by Panel (B) of Table 10. However, our main conclusions remain unchanged: eliminating uncertainty further raises the gains from trade, from 3.74% to 4.53%, and the interaction between uncertainty and economies of scale continues to be negative.

Nested Migration Choices. In our baseline model, workers' idiosyncratic abilities levels are independent across regions and sectors. As a robustness checks, we consider that workers draw abilities x_{mj} from a nested Fréchet distribution (Lind and Ramondo 2023), with correlation between sectors within a region being χ. Thus, the equilibrium allocation can be expressed as

$$\lambda_{mj} = \frac{U_{mj}^{\frac{\kappa}{(1-\gamma)(1-\chi)}}}{\sum_{j'} U_{mj'}^{\frac{\kappa}{(1-\gamma)(1-\chi)}}} \frac{\left(\sum_{j'} U_{mj'}^{\frac{\kappa}{(1-\gamma)(1-\chi)}}\right)^{1-\chi}}{\sum_{m'} \left(\sum_{j'} U_{m'j'}^{\frac{\kappa}{(1-\gamma)(1-\chi)}}\right)^{1-\chi}}. \tag{51}$$

This expression is isomorphic to the allocation obtained in a sequential choice framework, in which workers first select their region of residence and then choose their sector of employment.

Given the limited availability of estimates for the correlation in individual productivity across sectors, we set $\chi = 0.2$, which is consistent with the slight positive correlation in individual productivity between agricultural and nonagricultural activities documented by Lagakos and Waugh (2013).

As shown in Panel (C) of Table 10, our baseline findings are robust to this modification. If anything, the interaction between uncertainty and economies of scale is slightly amplified, driven by the higher elasticity of substitution across sectors.

Region-sector Productivity Shocks. Our model abstracts from productivity shocks that are specific to each subnational region-sector pair within the United States, which may lead us to underestimate the extent of uncertainty. To address this limitation, we introduce a productivity shock for each CZ-sector pair. Specifically, we assume that productivity shocks are drawn independently across regions and sectors from a log-normal distribution with log productivity distributed as $\mathcal{N}(-\tilde{\sigma}_j^2/2, \tilde{\sigma}_j^2)$, where the variance $\tilde{\sigma}_j^2$ is allowed to vary across sectors (agriculture, mining, manufacturing, and services). We calibrate $\tilde{\sigma}_j^2$ so that the model-generated data match the empirical R^2 values from regressions of CZ-level sectoral growth in real GDP per worker on sector-by-year fixed effects.[17]

Panel (D) of Table 10 reports the gains from trade under this specification. We find that introducing region-sector-specific productivity shocks leaves our quantitative results largely unchanged. The reason is that idiosyncratic shocks across regions and sectors

[17]Using county GDP and employment data available for the period 2001-2022, aggregated by CZs, the resulting R^2 values are 0.08, 0.42, 0.09, and 0.30 for agriculture, mining, manufacturing, and services, respectively.

largely average out at the national level, while country-sector-specific shocks remain the primary source of aggregate changes in volatility and specialization following trade liberalization.

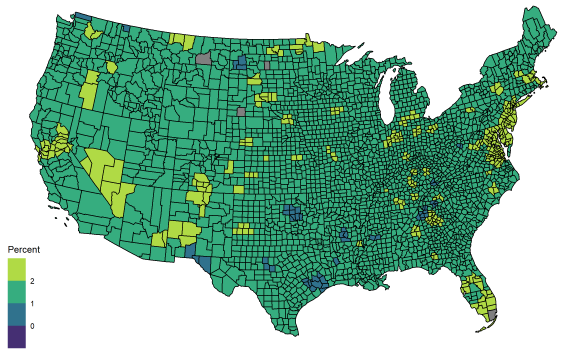
8 Conclusion

In this paper, we construct a multi-sector spatial equilibrium model, featuring economies of scale, volatility in productivity, and heterogeneous workers who choose the region and sector where to work. We use the model to study how uncertainty affects specialization patterns and welfare. We provide analytical characterizations of the interactions between specialization, economies of scale, and uncertainty. We find empirical support for the model predictions using panel data for U.S. commuting zones. Quantitatively, the model calibrated to match features of subnational U.S. economies and other countries, for 2017, predicts that volatility shifts employment away from sectors and regions with high real wage volatility, and that considering economies of scale and uncertainty together cuts the gains from trade for the United States by about 20 percent.

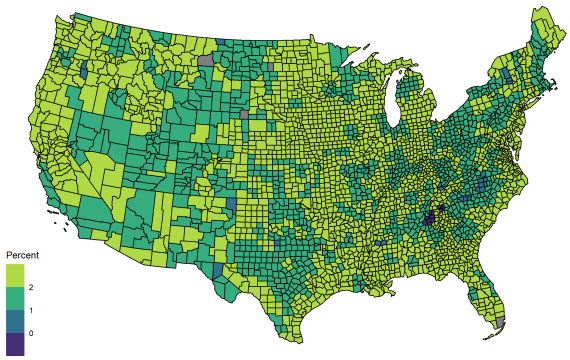
Our analysis highlights that volatility in aggregate productivity shapes specialization patterns within and across countries, as long as workers are risk averse. In principle, volatility can manifest in various other forms, such as trade policy uncertainty (Handley and Limão 2017, Alessandria et al. 2025) and supply chain disruptions (Grossman et al. 2023). Additionally, volatility may alter firm responses, such as increasing the inaction area of investments (Das, Roberts and Tybout 2007). Exploring the effects of these other types of volatility on specialization patterns and their interaction with economies of scale presents an intriguing area for future research.

Figure 8: Regional Gains from Trade.

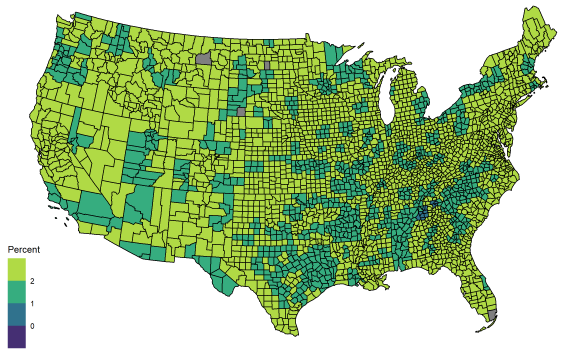
(a) Baseline (avg 1.74%)



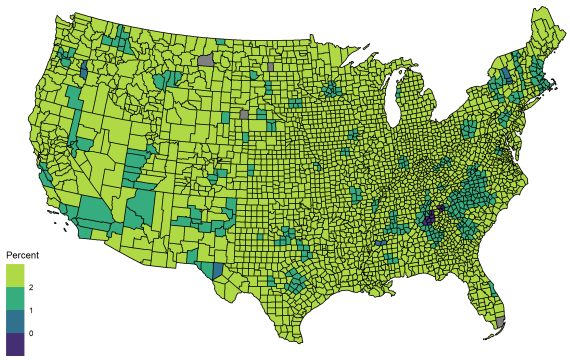
(b) No scale economies (avg 2.04%)



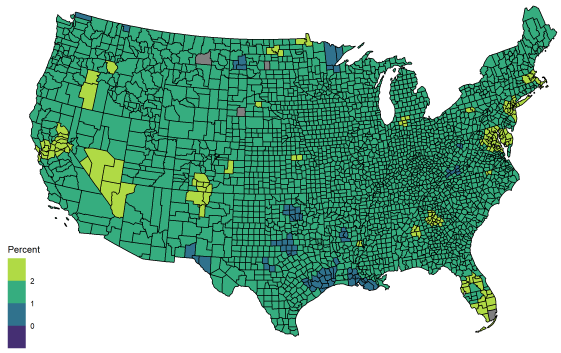
(c) No uncertainty (avg 2.17%)



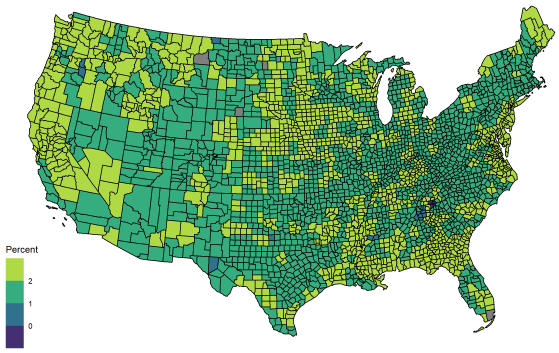
(d) No uncertainty & no scale economies (avg 2.38%)



(e) $\kappa = 5$ (avg 1.65%)



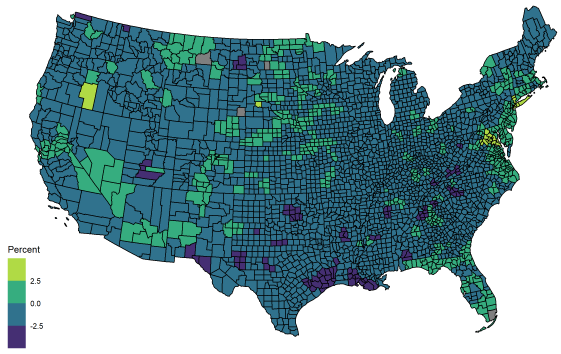
(f) $\kappa = 5$ & no scale economies (avg 2.00%)



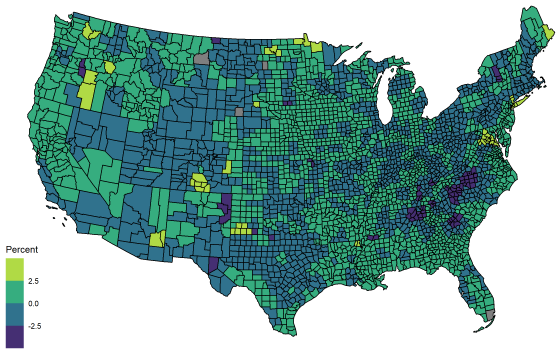
Note: Gains from trade in each CZ, $\sum_j (\lambda_{mj} / \sum_k \lambda_{mk}) \hat{U}_{mj}^{\frac{1}{1-\gamma}}$. For the model without uncertainty, we set $\sigma_{i,j} = 0$, for all i, j. For the model with no scale effects, we set $\rho_j = 0$, for all j. For panels (a)-(d), $\kappa = 3$.

Figure 9: Employment Reallocation From Autarky.

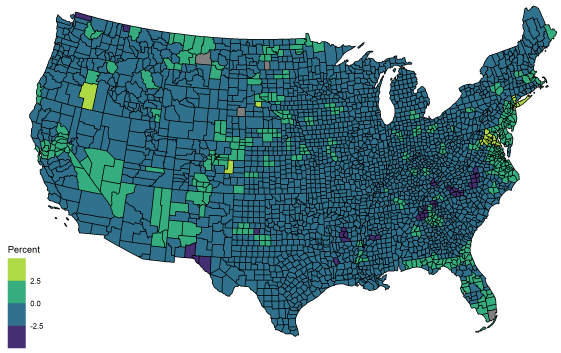
(a) Baseline (avg -0.63%)



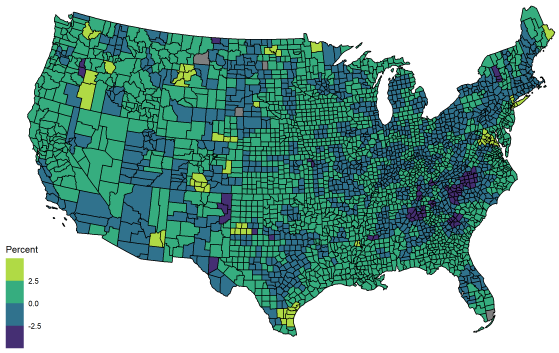
(b) No scale economies (avg 0.09%)



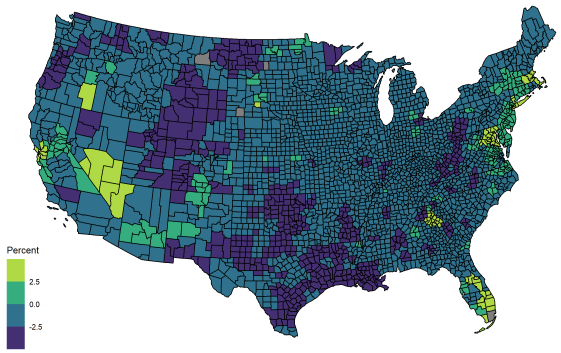
(c) No uncertainty (avg -0.50%)



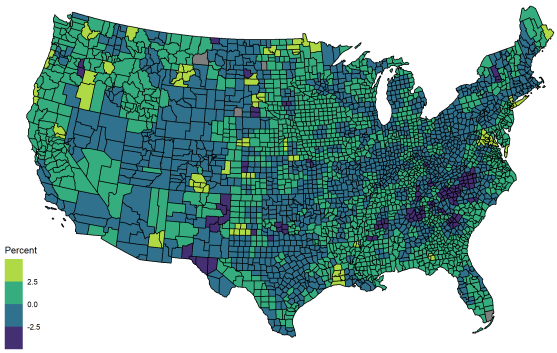
(d) No uncertainty & no scale economies (0.19%)



(e) $\kappa = 5$ (avg -1.36%)



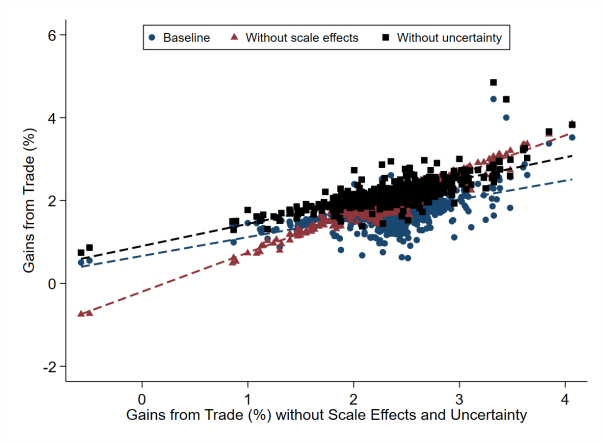
(f) $\kappa = 5$ & no scale economies (avg 0.11%)



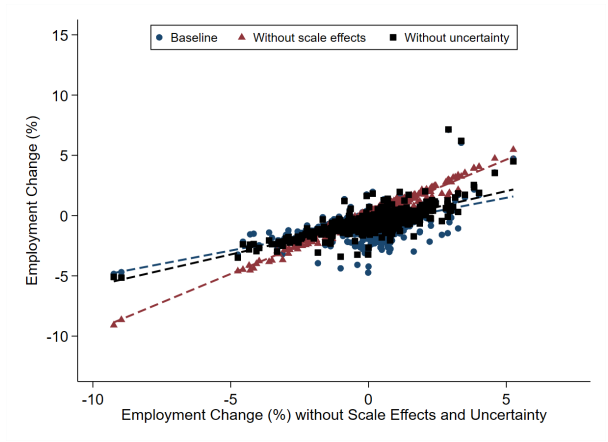
Note: Proportional changes in employment from autarky to the baseline equilibrium, in each CZ. For the model with no uncertainty, we set $\sigma_{i,j} = 0$ for all i, j. For the model with no scale effects, we set $\rho_j = 0$, for all j. For panels (a)-(d), $\kappa = 3$. Averages calculated over values across CZ's.

Figure 10: Regional Gains from Trade and Employment Reallocation: Correlations.

(a) Gains from trade



(b) Employment reallocation



Note: Regional gains from trade $\sum_j (\lambda_{mj} / \sum_k \lambda_{mk}) \hat{U}_{mj}^{\frac{1}{1-\gamma}}$ and proportional changes in employment from autarky to the equilibrium calculated with baseline calibration, no scale economies ($\rho_j = 0$, for all j), no uncertainty ($\sigma_{i,j} = 0$, for all i, j), and no scale economies and uncertainty.

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A Derivations and Proofs

We denote by: $\bar{X}$ the expected value of a state-dependent variable X, $\bar{X} \equiv \sum_s \mathbb{P}(s)X(s)$; X^* the value of X in the deterministic equilibrium; $\hat{X}(s) \equiv dX(s)/X^*$ deviations around X^* for state-dependent variables ($\hat{X} \equiv dX/X^*$ for non-state dependent variables); and $\hat{\bar{X}} \equiv \sum_s \mathbb{P}(s)\hat{X}(s)$. We normalize global expenditure in each state s to one.

A.1 Proof of Lemma 1

Assume that trade is frictionless across regions and countries. The market clearing condition for sector j and region m is

$$W_{mj}(s)L_{mj} = \frac{(W_{mj}(s)/A_{mj}(s))^{-\theta_j}}{\sum_{m'}(W_{m'j}(s)/A_{m'j}(s))^{-\theta_j}}\alpha_j. \tag{A.1}$$

We can solve for wages analytically,

$$W_{mj}(s) = \alpha_j \frac{A_{mj}(s)^{\frac{\theta_j}{\theta_j+1}} L_{mj}^{-\frac{1}{\theta_j+1}}}{\sum_{m'} A_{m'j}(s)^{\frac{\theta_j}{\theta_j+1}} L_{m'j}^{\frac{\theta_j}{\theta_j+1}}}, \tag{A.2}$$

as well as trade shares,

$$\pi_{mj}(s) = \frac{(A_{mj}(s)L_{mj})^{\frac{\theta_j}{\theta_j+1}}}{\sum_{m'}(A_{m'j}(s)L_{m'j})^{\frac{\theta_j}{\theta_j+1}}}$$

Using (6), we can further solve for the final-good price index

$$P_m(s) = C\prod_j \left(\sum_{m'\in M} A_{m'j}(s)^{\frac{\theta_j}{\theta_j+1}} L_{m'j}^{\frac{\theta_j}{\theta_j+1}}\right)^{-\alpha_j\frac{1+\theta_j}{\theta_j}}, \tag{A.3}$$

where $C \equiv \prod_j (C_j)^{\alpha_j} > 0$ and $C_j \equiv \Gamma\left(1-\frac{\eta_j-1}{\theta_j}\right)^{\frac{1}{\eta_j-1}}$.

Taking logs on (A.2) and (A.3) yields

$$\log W_{mj}(s) = \log\alpha_j + \frac{\theta_j}{\theta_j+1}\log A_{mj}(s) - \frac{1}{\theta_j+1}\log L_{mj} - \log\sum_{m'} A_{m'j}(s)^{\frac{\theta_j}{\theta_j+1}} L_{m'j}^{\frac{\theta_j}{\theta_j+1}},$$

and

$$\log P_m(s) = \sum_j \alpha_j \log C_j - \sum_j \alpha_j \frac{1+\theta_j}{\theta_j}\log\sum_{m'} A_{m'j}(s)^{\frac{\theta_j}{\theta_j+1}} L_{m'j}^{\frac{\theta_j}{\theta_j+1}}.$$

We calculate fluctuations of wages and prices around their expected values by linearizing

the two previous expressions (we omit $\hat{L}$ since they do not affect variances),

$$\hat{W}_{mj}(s) = \frac{\theta_j}{\theta_j + 1}\hat{A}_{mj}(s) - \sum_{m'} \bar{\pi}_{m'j}\frac{\theta_j}{\theta_j + 1}\hat{A}_{m'j}(s), \tag{A.4}$$

and

$$\hat{P}_m(s) = -\sum_k \alpha_k \sum_{m'} \bar{\pi}_{m'k}\hat{A}_{m'k}(s), \tag{A.5}$$

where $\bar{\pi}_{i,j} \equiv \sum_{m \in M_i} \bar{\pi}_{mj}$, and

$$\hat{A}_{mj}(s) = \hat{A}_{i,j}(s)$$

for all $m \in M_i$.

Hence,

$$\begin{aligned}\hat{W}_{mj}(s) - \hat{P}_m(s) = &\left(\frac{\theta_j}{\theta_j + 1}(1 - \bar{\pi}_{i,j}) + \alpha_j \bar{\pi}_{i,j}\right)\hat{A}_{i,j}(s) - \left(\frac{\theta_j}{\theta_j + 1} - \alpha_j\right)\sum_{i' \neq i} \bar{\pi}_{i',j}\hat{A}_{i',j}(s) \\ &+ \sum_{k \neq j}\sum_{i'} \alpha_k \bar{\pi}_{i',k}\hat{A}_{i',k}(s).\end{aligned}$$

With i.i.d. shocks, $\Sigma_{ij,i'k} = 0$ for all $i \neq i'$ and $j \neq k$ so that

$$\Xi_{i,j} = \left(\frac{\theta_j}{\theta_j + 1}(1 - \bar{\pi}_{i,j}) + \alpha_j \bar{\pi}_{i,j}\right)^2 \sigma^2_{i,j} + \left(\frac{\theta_j}{\theta_j + 1} - \alpha_j\right)^2 \sum_{i' \neq i} (\bar{\pi}_{i',j})^2 \sigma^2_{i',j} + \sum_{k \neq j}\sum_{i'} \alpha_k^2 (\bar{\pi}_{i',k})^2 \sigma^2_{i',k}.$$

Under autarky, $\bar{\pi}_{i,j} = 1$ and $\bar{\pi}_{i',j} = 0$ for all $i' \neq i$. Then, the first term becomes $\alpha_j^2 \sigma^2_{i,j}$ and all the cross-country terms vanish,

$$\Xi^A_{i,j} = \sum_j \alpha_j^2 \sigma^2_{i,j}.$$

A.2 Autarky

Using (A.1) and (A.3) under international trade autarky, but frictionless trade across regions within a country, the autarky wage is given by $W^A_{i,j} = \alpha_j / L^A_{i,j}$, while the price index is $P^A_i(s) = C \prod_j \left(A_{i,j}(s) L^A_{i,j}\right)^{-\alpha_j}$. Hence, the average real wage is given by

$$\frac{\bar{W}^A_{i,j}}{\bar{P}^A_i} = \frac{\alpha_j}{C L^A_{i,j}} \mathbb{E}\left[\prod_{j'} (L^A_{i,j'} A_{i,j'}(s))^{\alpha_{j'}}\right],$$

Using (2), labor shares in country i and sector j under autarky are

$$\lambda^A_{i,j} = \frac{\left(\bar{W}^A_{i,j} / \bar{P}^A_i\right)^\kappa}{\sum_{j'} \left(\bar{W}^A_{i,j'} / \bar{P}^A_i\right)^\kappa}$$

With $L_{i,j}^A = C_1(\lambda_{i,j}^A)^{\frac{\kappa-1}{\kappa}} L_i$, we get that $\lambda_{i,j}^A = \alpha_j$, and $W_{i,j}^A = C_1^{-1}\alpha_j^{1/\kappa}/L_i$.

A.3 Proof of Proposition 1

A second-order approximation of expected utility in (1) yields

$$U_{mj} \approx \frac{1}{1-\gamma}\left(\frac{\bar{W}_{mj}}{\bar{P}_m}\right)^{1-\gamma}\left(1 - \frac{\gamma(1-\gamma)}{2}\Xi_{mj}\right), \tag{A.6}$$

with fluctuations around the deterministic equilibrium yielding

$$\hat{U}_{mj} \approx (1-\gamma)(\hat{\bar{W}}_{mj} - \hat{\bar{P}}_m) - \frac{\gamma(1-\gamma)}{2} d\Xi_{mj}, \tag{A.7}$$

where we use (A.4), (A.5), and Assumption 1 to calculate

$$\hat{\bar{W}}_{mj} = -\frac{1}{\theta_j+1}(1-\rho_j\theta_j)\hat{L}_{mj} - \frac{\theta_j}{\theta_j+1}(\rho_j+1)\sum_n \pi_{nj}^* \hat{L}_{nj} \tag{A.8}$$

$$\hat{\bar{P}}_m = -\sum_k \alpha_k \sum_n \pi_{nk}^*(1+\rho_k)\hat{L}_{nk}, \tag{A.9}$$

with the approximation taken around the deterministic equilibrium.

Using the labor-supply condition in (2), employment shares satisfy

$$\hat{\lambda}_{mj} = \frac{\kappa}{1-\gamma}\hat{U}_{mj} - \sum_{n\in M_i}\sum_k \lambda_{nk}^* \frac{\kappa}{1-\gamma}\hat{U}_{nk}. \tag{A.10}$$

Under frictionless trade within country i, workers in the same industry earn the same expected real wage across regions, so $\hat{L}_{mj} = \hat{L}_{nj}$ and $\hat{\lambda}_{mj} = \hat{\lambda}_{nj}$ for all $m, n \in M_i$. Using (4),

$$\hat{L}_{mj} = \frac{\kappa-1}{\kappa}\hat{\lambda}_{mj}.$$

Moreover, since country i is small, $\hat{L}_{nj} = 0$ for all $n \notin M_i$. Hence, from (A.8),

$$\hat{\bar{W}}_{mj} = \left[\frac{1}{1+\theta_j}(\rho_j\theta_j - 1) - \pi_{i,j}^* \frac{\theta_j}{1+\theta_j}(\rho_j+1)\right]\frac{\kappa-1}{\kappa}\hat{\lambda}_{mj}. \tag{A.11}$$

Substituting (A.7) and (A.11) into (A.10), and using

$$\sum_{n\in M_i}\sum_k \lambda_{nk}^*\hat{\lambda}_{nk} = 0,$$

gives[18]

$$\hat{\lambda}_{mj} = \frac{\gamma\kappa}{2(1-\xi_j)} \left[\sum_k \lambda^*_{i,k}\, d\Xi_{mk} - d\Xi_{mj} \right], \tag{A.12}$$

where

$$\xi_j \equiv -(\kappa-1)\frac{1-\theta_j\rho_j + \pi^*_{i,j}\theta_j(\rho_j+1)}{1+\theta_j} < 0,$$

because $\theta_j\rho_j < 1$.

Now consider an increase in sector-j volatility in country i, holding all other variances fixed. By Lemma 1,

$$d\Xi_{mj} = \left(\frac{\theta_j}{1+\theta_j}(1-\pi^*_{i,j}) + \alpha_j\pi^*_{i,j} \right)^2 d\sigma^2_{i,j},$$

whereas for every $k \neq j$,

$$d\Xi_{mk} = (\alpha_j\pi^*_{i,j})^2 d\sigma^2_{i,j}.$$

Substituting into (A.12) and using

$$\sum_{k\neq j} \lambda^*_{i,k} = 1 - \lambda^*_{i,j},$$

we obtain

$$\hat{\lambda}_{mj} = -\frac{\gamma\kappa(1-\lambda^*_{i,j})}{2(1-\xi_j)} \frac{\theta_j}{1+\theta_j}(1-\pi^*_{i,j}) \left[\frac{\theta_j}{1+\theta_j}(1-\pi^*_{i,j}) + 2\alpha_j\pi^*_{i,j} \right] d\sigma^2_{i,j}. \tag{A.13}$$

Since all terms in brackets are positive and $1-\zeta_j > 0$, it follows that

$$\frac{\partial(\lambda_{mj}/\lambda^*_{mj})}{\partial\sigma^2_{i,j}} < 0.$$

Expression (A.13) immediately shows that the magnitude of the effect is increasing in γ. In addition, since ξ_j is increasing in ρ_j, stronger industry-level economies of scale reduce $1-\xi_j$ and therefore amplify the negative effect of volatility,

$$\frac{\partial^2(\lambda_{mj}/\lambda^*_{mj})}{\partial\sigma^2_{i,j}\partial\rho_j} < 0, \qquad \frac{\partial^2(\lambda_{mj}/\lambda^*_{mj})}{\partial\sigma^2_{i,j}\partial\gamma} < 0.$$

Finally, differentiating (A.13) with respect to $\pi^*_{i,j}$ shows that the effect becomes less negative as $\pi^*_{i,j}$ rises, provided that

$$\alpha_j < \frac{\theta_j}{1+\theta_j},$$

[18] When ζ_k is similar across sectors, the term is approximately zero: $\sum_{n\in M_i}\sum_k \lambda^*_{nk}\hat{\bar{W}}_{nk} = \frac{1}{\kappa}\sum_{n\in M_i}\sum_k \zeta_k\lambda^*_{nk}\hat{\lambda}_{nk} \approx 0$.

or equivalently,

$$\frac{1}{\theta_j} < \frac{1-\alpha_j}{\alpha_j}.$$

Hence,

$$\frac{\partial^2(\lambda_{mj}/\lambda^*_{mj})}{\partial\sigma^2_{i,j}\partial\pi^*_{i,j}} > 0.$$

Since lower $\pi^*_{i,j}$ corresponds to greater trade exposure, more trade exposure amplifies the negative impact of volatility.

A.4 Average Expected Utility

Average expected utility for workers in region m and industry j is the same across regions and sectors in a country,

$$\begin{aligned}
\mathbb{U}_{mj} &= \int_0^\infty U_{mj} x^{1-\gamma} g(x) \prod_{j'\neq j \text{ or } m'\neq m \ (m'\in M_i)} G\left(\frac{U_{mj}^{\frac{1}{1-\gamma}} x}{U_{m'j'}^{\frac{1}{1-\gamma}}}\right) \frac{dx}{\lambda_{mj}} \\
&= \int_0^\infty U_{mj} y^{-\frac{1-\gamma}{\kappa}} \left(\sum_{m'\in M_i, j'} \left(\frac{U_{m'j'}}{U_{mj}}\right)^{\frac{\kappa}{1-\gamma}}\right)^{\frac{1-\gamma}{\kappa}} \exp(-y) dy \qquad \text{(A.14)} \\
&= \Gamma\left(1-\frac{1-\gamma}{\kappa}\right) \left(\sum_{m'\in M_i, j'} (U_{m'j'})^{\frac{\kappa}{1-\gamma}}\right)^{\frac{1-\gamma}{\kappa}} = \mathbb{U}_i,
\end{aligned}$$

where $\Gamma(\cdot)$ is the Gamma function, and $G(\cdot)$ is a unit-Fréchet distribution with shape parameter $\kappa > 0$ and density $g(\cdot)$. The first equality follows from the definition of average utility for workers in region m and industry j, which is the integral of utility conditional on choosing region m and industry j, adjusted by the probability of choosing region m and industry j. The second equality uses the change of variables $y = \sum_{m'\in M_i, j'} \left(\frac{U_{m'j'}}{U_{mj}}\right)^{\frac{\kappa}{1-\gamma}} x^{-\kappa}$, and the final line simplifies the formula.

A.5 Proof of Proposition 2

From (18),

$$\mathbb{U}_i^{\frac{1}{1-\gamma}} = \Gamma\left(\frac{\kappa+\gamma-1}{\kappa}\right)^{\frac{1}{1-\gamma}} \left(\sum_j U_{i,j}^{\frac{\kappa}{1-\gamma}}\right)^{\frac{1}{\kappa}}.$$

Applying the second-order approximation[19]

$$U_{i,j} \approx \frac{1}{1-\gamma}\left(\frac{\bar{W}_{i,j}}{\bar{P}_i}\right)^{1-\gamma}\left(1-\frac{\gamma(1-\gamma)}{2}\Xi_{i,j}\right),$$

we obtain

$$\mathbb{U}_i^{\frac{1}{1-\gamma}} \approx C\left(\sum_j\left(\frac{\bar{W}_{i,j}}{\bar{P}_i}\right)^{\kappa}\left(1-\frac{\gamma(1-\gamma)}{2}\Xi_{i,j}\right)^{\frac{\kappa}{1-\gamma}}\right)^{\frac{1}{\kappa}},$$

for a constant $C>0$. Using the expression for labor shares,

$$\lambda_{i,j}=\frac{(\bar{W}_{i,j}/\bar{P}_i)^{\kappa}\left(1-\frac{\gamma(1-\gamma)}{2}\Xi_{i,j}\right)^{\frac{\kappa}{1-\gamma}}}{\sum_{j'}(\bar{W}_{i,j'}/\bar{P}_i)^{\kappa}\left(1-\frac{\gamma(1-\gamma)}{2}\Xi_{i,j'}\right)^{\frac{\kappa}{1-\gamma}}},$$

it follows that

$$\mathbb{U}_i^{\frac{1}{1-\gamma}} \approx C_1\prod_j\left(1-\frac{\gamma(1-\gamma)}{2}\Xi_{i,j}\right)^{\frac{\alpha_j}{1-\gamma}}\prod_j\lambda_{i,j}^{-\frac{\alpha_j}{\kappa}}\frac{\prod_j\bar{W}_{i,j}^{\alpha_j}}{\bar{P}_i}.$$

Next, using

$$\bar{P}_i=C_2\prod_j\left(\left(\frac{\bar{W}_{i,j}}{\bar{A}_{i,j}}\right)^{-\theta_j}\left(\bar{\pi}_{ii}^{j}\right)^{-1}\right)^{-\frac{\alpha_j}{\theta_j}},$$

together with $\bar{A}_{i,j}\propto L_{i,j}^{\rho_j}$ and $L_{i,j}\propto\lambda_{i,j}^{(\kappa-1)/\kappa}$, yields

$$\frac{\prod_j\bar{W}_{i,j}^{\alpha_j}}{\bar{P}_i}=C_3\prod_j(\bar{\pi}_{ii}^{j})^{-\frac{\alpha_j}{\theta_j}}\prod_j\lambda_{i,j}^{\alpha_j\rho_j}\prod_j\lambda_{i,j}^{-\frac{\alpha_j\rho_j}{\kappa}}.$$

Taking the ratio with the analogous autarky expression and using $\lambda_{i,j}^{A}=\alpha_j$ give (22).

A.6 Proof of Lemma 2

For workers without access to AD securities, $C_{mj}^{H}(s)=W_{mj}(s)$, so $\Xi_{i,j}^{H}=\Xi_{i,j}$ follows from the proof of Lemma 1.

For workers with access, log-linearizing (25) around the deterministic equilibrium gives $\hat{C}_{mj}^{AD}(s)-\hat{P}_m(s)=-(\hat{Q}_i(s)+\hat{P}_m(s))/\gamma$. With frictionless trade, $\hat{P}_m(s)=\hat{P}_i(s)$ for all m, and log-linearizing (32) and (33) yields

$$\hat{Q}_i(s)=-\gamma\hat{Y}^{AD}(s)+(\gamma-1)\hat{P}_i(s),$$

[19]When country i contains multiple regions, frictionless trade ensures that, for each sector j, relative wages across regions remain fixed, and final-good prices are equalized across regions. As a result, the multiple regions within country i can be aggregated into a single representative region without loss of generality.

where $Y^{AD}(s)$ is the income of insured workers in the world (global market) or in country i (national market). Hence,

$$\hat{C}^{AD}_{mj}(s) - \hat{P}_m(s) = \hat{Y}^{AD}(s) - \hat{P}_i(s),$$

which does not depend on j, so that $\Xi^{AD}_{i,j} = \text{var}(\hat{Y}^{AD}(s) - \hat{P}_i(s))$ is common across sectors. In the deterministic equilibrium, $\lambda^{AD*}_{mj} = \lambda^{H*}_{mj} = \lambda^*_{mj}$, so that $L^{AD}_{mj} = \mu_i L_{mj}$ and the sectoral composition of insured income in country i coincides with that of total income. Since wage deviations $\hat{W}_{mj}(s)$ are the same for all $m \in M_i$,

$$\hat{Y}^{AD}_i(s) = \hat{Y}_i(s) = \sum_k s_{ik}\hat{W}_{ik}(s) = \sum_k s_{ik}\frac{\theta_k}{\theta_k+1}\left[(1-\bar{\pi}_{i,k})\hat{A}_{i,k}(s) - \sum_{i'\neq i}\bar{\pi}_{i',k}\hat{A}_{i',k}(s)\right], \tag{A.15}$$

where s_{ik} is the share of country i's income from sector k, and the expression for $\hat{W}_{ik}$ follows from (A.4).

Global AD securities. World income of insured workers is $Y^{AD}(s) = \sum_i \mu_i Y_i(s)$, so that $\hat{Y}^{AD}(s) = \sum_i \left(\mu_i \bar{Y}_i / \sum_n \mu_n \bar{Y}_n\right)\hat{Y}_i(s)$. If $\mu_i = \mu$ for all i, this is proportional to the deviation of world income, which is zero since world income is the numeraire in each state. Hence $\hat{Y}^{AD}(s) = 0$ and $\Xi^{AD}_{i,j} = \text{var}(\hat{P}_i(s))$. Using (A.5) and i.i.d. shocks, we obtain the first result in Lemma 2.

Country-level AD securities. Using (A.15) and (A.5),

$$\hat{Y}^{AD}_i(s) - \hat{P}_i(s) = \sum_k \left[\left(\frac{\theta_k}{1+\theta_k}s_{ik}(1-\bar{\pi}_{i,k}) + \alpha_k\bar{\pi}_{i,k}\right)\hat{A}_{i,k}(s) - \left(s_{ik}\frac{\theta_k}{1+\theta_k} - \alpha_k\right)\sum_{i'\neq i}\bar{\pi}_{i',k}\hat{A}_{i',k}(s)\right].$$

Taking the variance with i.i.d. shocks gives the second result in Lemma 2.

A.7 Proof of Proposition 3

We follow the steps in Appendix A.3. Both types earn the same expected wage per efficiency unit, $\bar{W}_{mj}$, and face the same expected price, $\bar{P}_m$. In the deterministic equilibrium the share of insured workers in each (m, j) is μ_i, so that (28) implies

$$\hat{L}_{mj} = \frac{\kappa-1}{\kappa}\left[\mu_i\hat{\lambda}^{AD}_{mj} + (1-\mu_i)\hat{\lambda}^{H}_{mj}\right].$$

Using (A.8), $\hat{L}_{mj} = \hat{L}_{nj}$ for all $m, n \in M_i$, and the small-country assumption, as in (A.11), we obtain

$$\hat{\bar{W}}_{mj} = \left[\frac{1}{1+\theta_j}(\rho_j\theta_j - 1) - \pi^*_{i,j}\frac{\theta_j}{1+\theta_j}(\rho_j+1)\right]\frac{\kappa-1}{\kappa}\left[\mu_i\hat{\lambda}^{AD}_{mj} + (1-\mu_i)\hat{\lambda}^{H}_{mj}\right] = \frac{\xi_j}{\kappa}\left[\mu_i\hat{\lambda}^{AD}_{mj} + (1-\mu_i)\hat{\lambda}^{H}_{mj}\right], \tag{A.16}$$

where $\xi_j \equiv -(\kappa-1)\frac{1-\theta_j\rho_j+\pi^*_{i,j}\theta_j(\rho_j+1)}{1+\theta_j} < 0$, as in Appendix A.3.

Workers with access to AD securities. Applying (A.10) and (A.7) to type AD, and noting that by Lemma 2 $d\Xi^{AD}_{mk}$ is common across all (m,k) in country i, as is $\hat{\tilde{P}}_m$, these terms cancel in relative employment shares. Using the same approximation as in Appendix A.3 for the average wage term,

$$\hat{\lambda}^{AD}_{mj} = \frac{\kappa}{1-\gamma}\hat{U}^{AD}_{mj} - \sum_{n\in M_i}\sum_k \lambda^*_{nk}\frac{\kappa}{1-\gamma}\hat{U}^{AD}_{nk} = \kappa\hat{\bar{W}}_{mj}. \tag{A.17}$$

Insured workers are fully insured against sectoral real-income volatility, so their employment shares respond only to expected real income. Substituting (A.17) into (A.16) and solving for $\hat{\bar{W}}_{mj}$,

$$\hat{\bar{W}}_{mj} = \frac{\xi_j(1-\mu_i)}{\kappa(1-\xi_j\mu_i)}\hat{\lambda}^H_{mj} = \frac{\tilde{\xi}_j}{\kappa}\hat{\lambda}^H_{mj}, \qquad \tilde{\xi}_j \equiv \frac{\xi_j(1-\mu_i)}{1-\xi_j\mu_i}, \tag{A.18}$$

so that

$$\hat{\lambda}^{AD}_{mj} = \kappa\hat{\bar{W}}_{mj} = \tilde{\xi}_j\hat{\lambda}^H_{mj}.$$

Workers without access to AD securities. Applying (A.10) and (A.7) to type H and using (A.18),

$$\hat{\lambda}^H_{mj} = \tilde{\xi}_j\hat{\lambda}^H_{mj} + \frac{\gamma\kappa}{2}\left[\sum_k \lambda^*_{i,k}\, d\Xi^H_{mk} - d\Xi^H_{mj}\right] \quad \Rightarrow \quad \hat{\lambda}^H_{mj} = \frac{\gamma\kappa}{2(1-\tilde{\xi}_j)}\left[\sum_k \lambda^*_{i,k}\, d\Xi^H_{mk} - d\Xi^H_{mj}\right].$$

Since $\Xi^H_{mk} = \Xi_{mk}$ (Lemma 2), the same substitution of $d\Xi_{mj}$ and $d\Xi_{mk}$ as in Appendix A.3 gives

$$\hat{\lambda}^H_{mj} = -\frac{\gamma\kappa(1-\lambda^*_{i,j})}{2(1-\tilde{\xi}_j)}\frac{\theta_j}{1+\theta_j}(1-\pi^*_{i,j})\left[\frac{\theta_j}{1+\theta_j}(1-\pi^*_{i,j}) + 2\alpha_j\pi^*_{i,j}\right] d\sigma^2_{i,j}. \tag{A.19}$$

Because $1-\tilde{\xi}_j = (1-\xi_j)/(1-\xi_j\mu_i)$, comparing with (A.13) yields

$$\hat{\lambda}^H_{mj} = (1-\xi_j\mu_i)\hat{\lambda}^B_{mj}, \qquad \hat{\lambda}^{AD}_{mj} = \tilde{\xi}_j\hat{\lambda}^H_{mj} = \xi_j(1-\mu_i)\hat{\lambda}^B_{mj},$$

where $\hat{\lambda}^B_{mj}$ is the baseline response in (A.13), and

$$\hat{\lambda}_{mj} \equiv \mu_i\hat{\lambda}^{AD}_{mj} + (1-\mu_i)\hat{\lambda}^H_{mj} = \left[\mu_i\xi_j(1-\mu_i) + (1-\mu_i)(1-\xi_j\mu_i)\right]\hat{\lambda}^B_{mj} = (1-\mu_i)\hat{\lambda}^B_{mj}.$$

Signs and comparative statics. Since $\xi_j < 0$ and $\hat{\lambda}^B_{mj} < 0$, $\hat{\lambda}^H_{mj} < 0$, $\hat{\lambda}^{AD}_{mj} \geq 0$, and $\hat{\lambda}_{mj} \leq 0$, with equality for $\mu_i = 1$. Moreover,

$$\frac{\partial\hat{\lambda}^H_{mj}}{\partial\mu_i} = -\xi_j\hat{\lambda}^B_{mj} < 0,$$

and since $\hat{\lambda}^B_{mj}$ is proportional to $d\sigma^2_{i,j}$, $\partial^2\hat{\lambda}^H_{mj}/\partial\sigma^2_{i,j}\partial\mu_i < 0$. Equivalently, $\partial\tilde{\xi}_j/\partial\mu_i = \xi_j(\xi_j - 1)/(1-\xi_j\mu_i)^2 > 0$ and $\partial\hat{\lambda}^H_{mj}/\partial\tilde{\xi}_j < 0$.

Finally, the comparative statics in Proposition 1 carry over to $\hat{\lambda}^H_{mj}$. The term $(1-\lambda^*_{i,j})\frac{\theta_j}{1+\theta_j}(1-\pi^*_{i,j})[\cdot]$ is unchanged, and $\tilde{\xi}_j$ is increasing in ξ_j, $\partial\tilde{\xi}_j/\partial\xi_j = (1-\mu_i)/(1-\xi_j\mu_i)^2 \geq 0$, so $\tilde{\xi}_j$ moves with ρ_j and $\pi^*_{i,j}$ in the same direction as ξ_j. Hence the effect is amplified by γ and ρ_j, and dampened by $\pi^*_{i,j}$ if $\alpha_j < \theta_j/(1+\theta_j)$. The same holds for the total response $\hat{\lambda}_{mj} = (1-\mu_i)\hat{\lambda}^B_{mj}$.

A.8 Proof of Proposition 4

Following the approach in Appendix A.5, we derive the gains from trade separately for workers with access to AD securities and those without. Both types face the same expected real wage, $\bar{W}_{i,j}/\bar{P}_i$, and therefore the same price term. Productivity, however, depends on total employment through economies of scale, $\bar{A}_{i,j} \propto L^{\rho_j}_{i,j}$, with $L_{i,j} \propto \mu_i(\lambda^{AD}_{i,j})^{1-1/\kappa} + (1-\mu_i)(\lambda^{H}_{i,j})^{1-1/\kappa}$ by (28). Applying the second-order approximation of $U^F_{i,j}$ with $\Xi^F_{i,j}$, and the expression for type-specific labor shares (27), yields

$$\frac{\mathbb{U}^{AD}_i}{\mathbb{U}^{AD,A}_i} \approx \prod_j \left(\bar{\pi}^j_{i,i}\right)^{-\frac{(1-\gamma)\alpha_j}{\theta_j}} \prod_j \left(\frac{\mu_i(\lambda^{AD}_{i,j})^{1-1/\kappa} + (1-\mu_i)(\lambda^{H}_{i,j})^{1-1/\kappa}}{\mu_i(\lambda^{AD,A}_{i,j})^{1-1/\kappa} + (1-\mu_i)(\lambda^{H,A}_{i,j})^{1-1/\kappa}}\right)^{(1-\gamma)\alpha_j\rho_j} \prod_j \left(\frac{\lambda^{AD}_{i,j}}{\lambda^{AD,A}_{i,j}}\right)^{-\frac{(1-\gamma)\alpha_j}{\kappa}} \times \prod_j \left(\frac{1-\frac{(1-\gamma)\gamma}{2}\Xi^{AD}_{i,j}}{1-\frac{(1-\gamma)\gamma}{2}\Xi^{AD,A}_{i,j}}\right)^{\alpha_j}, \tag{A.20}$$

$$\frac{\mathbb{U}^{H}_i}{\mathbb{U}^{H,A}_i} \approx \prod_j \left(\bar{\pi}^j_{i,i}\right)^{-\frac{(1-\gamma)\alpha_j}{\theta_j}} \prod_j \left(\frac{\mu_i(\lambda^{AD}_{i,j})^{1-1/\kappa} + (1-\mu_i)(\lambda^{H}_{i,j})^{1-1/\kappa}}{\mu_i(\lambda^{AD,A}_{i,j})^{1-1/\kappa} + (1-\mu_i)(\lambda^{H,A}_{i,j})^{1-1/\kappa}}\right)^{(1-\gamma)\alpha_j\rho_j} \prod_j \left(\frac{\lambda^{H}_{i,j}}{\lambda^{H,A}_{i,j}}\right)^{-\frac{(1-\gamma)\alpha_j}{\kappa}} \times \prod_j \left(\frac{1-\frac{(1-\gamma)\gamma}{2}\Xi^{H}_{i,j}}{1-\frac{(1-\gamma)\gamma}{2}\Xi^{H,A}_{i,j}}\right)^{\alpha_j}. \tag{A.21}$$

In autarky, $NX_i = B_i = 0$, nominal wages are constant across states, and all real-income uncertainty originates from final-good price fluctuations, which are common across sectors. Hence the autarkic equilibrium is the same for both types: $\lambda^{AD,A}_{i,j} = \lambda^{H,A}_{i,j} = \lambda^{A}_{i,j} = \alpha_j$, $\Xi^{AD,A}_{i,j} = \Xi^{H,A}_{i,j} = \Xi^{A}_{i,j}$, and $\mathbb{U}^{AD,A}_i = \mathbb{U}^{H,A}_i = \mathbb{U}^{A}_i$. Therefore,

$$\frac{\mathbb{U}_i(\mu_i)}{\mathbb{U}^A_i} = \mu_i\frac{\mathbb{U}^{AD}_i}{\mathbb{U}^A_i} + (1-\mu_i)\frac{\mathbb{U}^{H}_i}{\mathbb{U}^A_i}.$$

Factoring out the terms common to both types and raising to the power $1/(1-\gamma)$ gives (42). With $\mu_i = 0$, the mixture term equals $(\lambda^H_{i,j}/\lambda^A_{i,j})^{1-1/\kappa}$ and (42) collapses to (22).

A.9 Proof of Corollary 1

Setting $\mu_i = 1$ in (42) gives the gains from trade under full insurance,

$$GT_i(1) \approx \prod_j \left(\frac{\lambda_{i,j}^{AD}}{\lambda_{i,j}^{A}}\right)^{-\frac{\alpha_j}{\kappa}} \prod_j \left(\bar{\pi}_{i,i}^{j}\right)^{-\frac{\alpha_j}{\theta_j}} \prod_j \left(\frac{\lambda_{i,j}^{AD}}{\lambda_{i,j}^{A}}\right)^{\alpha_j\rho_j} \prod_j \left(\frac{\lambda_{i,j}^{AD}}{\lambda_{i,j}^{A}}\right)^{-\frac{\alpha_j\rho_j}{\kappa}} \prod_j \left(\frac{1-\frac{(1-\gamma)\gamma}{2}\Xi_{i,j}^{AD}}{1-\frac{(1-\gamma)\gamma}{2}\Xi_{i,j}^{A}}\right)^{\frac{\alpha_j}{1-\gamma}}, \tag{A.22}$$

while setting $\mu_i = 0$ gives the baseline gains from trade $GT_i(0) = GT_i$ in (22), with $\lambda_{i,j}^{H} = \lambda_{i,j}$ and $\Xi_{i,j}^{H} = \Xi_{i,j}$. Since the autarkic equilibrium is the same in both economies, taking the ratio and holding $\bar{\pi}_{ii}^{j}$ fixed, the autarky terms and the domestic-trade-share terms cancel, and the three employment-share terms combine into $(\lambda_{i,j}^{AD}/\lambda_{i,j})^{\frac{\alpha_j}{\kappa}[(\kappa-1)\rho_j-1]}$, which gives (43).

B Model with Sectoral Input-Output Linkages

Each intermediate good ω is produced with labor and the aggregate sectoral good from each sector k, with shares δ_{mj}^{L} and δ_{mj}^{k}, respectively, with $\delta_{mj}^{L} + \sum_k \delta_{mj}^{k} = 1$. We present the equilibrium of the model with sectoral input-output links given the workers' location and sectoral choices. In this case, the unit cost of production in region m and sector j is:

$$c_{mj}(s) = \left(\frac{W_{mj}(s)}{\delta_{mj}^{L}}\right)^{\delta_{mj}^{L}} \prod_k \left(\frac{P_{mk}(s)}{\delta_{mj}^{k}}\right)^{\delta_{mj}^{k}}. \tag{B.1}$$

The price index in region m and sector j is

$$P_{mj}(s) = C_j \left(\sum_{l\in M} \left(\frac{\tau_{lm}^{j} c_{lj}(s)}{A_{lj}(s)}\right)^{-\theta_j}\right)^{-\frac{1}{\theta_j}}, \tag{B.2}$$

$C_j \equiv \Gamma\left(\frac{1-\eta_j+\theta_j}{\theta_j}\right)^{\frac{1}{1-\eta_j}}$, and the final-good price in region m is:

$$P_m(s) = \prod_j \left(\frac{P_{mj}(s)}{\alpha_j}\right)^{\alpha_j}. \tag{B.3}$$

The good-market clearing condition in region m and sector j requires that

$$Y_{lj}(s) = \sum_m \frac{(\tau_{lm}^{j} c_{lj}(s)/A_{lj}(s))^{-\theta_j}}{\sum_{l'\in M}(\tau_{l'm}^{j} c_{l'j}(s)/A_{l'j}(s))^{-\theta_j}} X_{mj}(s), \tag{B.4}$$

where $Y_{lj}(s)$ is the production value in region l and sector j, and $X_{mj}(s) = \sum_k \delta_{mk}^{j} Y_{mk}(s) + \alpha_j\left(\sum_k \delta_{mk}^{L} Y_{mk}(s)\right)$ is the total expenditures on goods from sector j in region m.

Finally, the labor market clearing requires that

$$\delta_{mj}^{L} Y_{mj}(s)/W_{mj}(s) = L_{mj}. \tag{B.5}$$

The left-hand side is the demand for labor in region m and sector j, and the right-hand side is the labor supply in region m and sector j.

We can combine (B.1)–(B.5) to solve for $\{W_{mj}(s), c_{mj}(s), Y_{mj}(s), P_{mj}(s), P_m(s)\}$. Finally, we can solve workers' location and sectoral choices λ_{mj} from (2).

With a slight abuse of notation, define $\widehat{x} \equiv \frac{x'}{x}$ as the counterfactual change in variable x. Next, we present the equilibrium system (B.1)–(B.5) in changes,

$$\widehat{c}_{mj}(s) = \left(\widehat{W}_{mj}(s)\right)^{\delta_{mj}^{L}} \prod_{k} \left(\widehat{P}_{mk}(s)\right)^{\delta_{mj}^{k}}, \tag{B.6}$$

$$\widehat{P}_{mj}(s) = \left(\sum_{l \in M} \pi_{lm}^{j}(s) \left(\frac{\hat{\tau}_{lm}^{j} \widehat{c}_{lj}(s)}{\widehat{A}_{lj}(s)}\right)^{-\theta_j}\right)^{-\frac{1}{\theta_j}}, \tag{B.7}$$

where $\widehat{A}_{lj}(s) = \widehat{L}_{lj}^{\rho_s}$ captures changes in economies of scale,

$$\widehat{P}_{m}(s) = \prod_{j} \left(\widehat{P}_{mj}(s)\right)^{\alpha_j}, \tag{B.8}$$

$$\widehat{Y}_{lj}(s) Y_{lj}(s) = \sum_{m} \frac{\pi_{lm}^{j}(s)(\hat{\tau}_{lm}^{j} \widehat{c}_{lj}(s)/\widehat{A}_{lj}(s))^{-\theta_j}}{\sum_{l' \in M} \pi_{l'm}^{j}(s)(\hat{\tau}_{l'm}^{j} \widehat{c}_{l'j}(s)/\widehat{A}_{l'j}(s))^{-\theta_j}} \widehat{X}_{mj}(s) X_{mj}(s), \tag{B.9}$$

where $\widehat{X}_{mj}(s)X_{mj}(s) = \sum_k \delta_{mk}^{j} \widehat{Y}_{mk}(s) Y_{mk}(s) + \alpha_j \left(\sum_k \delta_{mk}^{L} \widehat{Y}_{mk}(s) Y_{mk}(s)\right)$, and finally

$$\frac{\widehat{Y}_{mj}(s)}{\widehat{W}_{mj}(s)} = \widehat{L}_{mj}. \tag{B.10}$$

We can combine (B.6)–(B.10) to solve for $\{\widehat{W}_{mj}(s), \widehat{c}_{mj}(s), \widehat{Y}_{mj}(s), \widehat{P}_{mj}(s), \widehat{P}_{m}(s)\}$.

B.1 Derivation of Equation (48).

Using the expression in (B.1), the average unit cost of production in region m and industry j is

$$\bar{c}_{mj} = \left(\frac{\bar{W}_{mj}}{\delta_{mj}^{L}}\right)^{\delta_{mj}^{L}} \prod_{k} \left(\frac{\bar{P}_{mk}}{\delta_{mj}^{k}}\right)^{\delta_{mj}^{k}},$$

which combined with the expression for domestic trade shares,

$$\bar{\pi}_{mm}^{j} = C_j^{-\theta} \left(\frac{\bar{c}_{mj}/\bar{A}_{mj}}{\bar{P}_{mj}}\right)^{-\theta_j},$$

delivers the price index $\bar{P}_{mj}$.

Again, with some abuse of notation, let $\hat{x} \equiv \log(x'/x)$ denote the counterfactual change in variable x. Combining the changes in $\bar{c}_{mj}$ and $\bar{\pi}^j_{mm}$ yields

$$\hat{\bar{P}}_{mj} = \frac{\hat{\bar{\pi}}^j_{mm}}{\theta_j} - \hat{\bar{A}}_{mj} + \delta^L_{mj}\hat{\bar{W}}_{mj} + \sum_k \delta^k_{mj}\hat{\bar{P}}_{mk}.$$

After some matrix calculations, we get that

$$\hat{\bar{P}}_{mj} = \sum_k \zeta^k_{mj}\left(\frac{\hat{\bar{\pi}}^k_{mm}}{\theta_k} - \hat{\bar{A}}_{mk} + \delta^L_{mk}\hat{\bar{W}}_{mk}\right),$$

where ζ^k_{mj} is the (j,k) element of the Leontief inverse matrix $\left(\mathbf{I} - \{\delta^k_{mj}\}\right)^{-1}$. Applying this formula to autarky yields

$$\frac{\bar{P}^A_{mj}}{\bar{P}_{mj}} = \prod_k \left(\frac{\bar{\pi}^k_{mm}}{\bar{\pi}^{k,A}_{mm}}\right)^{-\frac{\zeta^k_{mj}}{\theta_k}} \prod_k \left(\frac{\lambda_{mk}}{\lambda^A_{mk}}\right)^{\frac{\kappa-1}{\kappa}\zeta^k_{mj}\rho_k} \prod_k \left(\frac{\bar{W}^A_{mk}}{\bar{W}_{mk}}\right)^{\zeta^k_{mj}\delta^L_{mk}},$$

where we used that $\hat{A}_{mj} = \frac{\kappa-1}{\kappa}\rho_j\hat{\lambda}_{mj}$. Further using that $\bar{P}_m = \prod_j (\bar{P}_{mj}/\alpha_j)^{\alpha_j}$ yields

$$\frac{\bar{W}_{mj}/\bar{P}_m}{\bar{W}^A_{mj}/\bar{P}^A_m} = \underbrace{\frac{\bar{\omega}_{mj}}{\bar{\omega}^A_{mj}}}_{\text{Workers Heterogeneity}} \times \underbrace{\prod_k \left(\frac{\bar{\pi}^k_{mm}}{\bar{\pi}^{k,A}_{mm}}\right)^{-\sum_j \frac{\alpha_j\zeta^k_{mj}}{\theta_k}}}_{\text{ACR}} \times \underbrace{\prod_k \left(\frac{\lambda_{mk}}{\lambda^A_{mk}}\right)^{\sum_j \alpha_j\zeta^k_{mj}\rho_k}}_{\text{Economies of scale}} \times \underbrace{\prod_k \left(\frac{\lambda_{mk}}{\lambda^A_{mk}}\right)^{-\frac{1}{\kappa}\sum_j \alpha_j\zeta^k_{mj}\rho_k}}_{\text{Interaction}},$$

with $\bar{\omega}_{mj} \equiv \bar{W}_{mj}/\prod_k \bar{W}_{mk}^{\sum_j \alpha_j\zeta^k_{mj}\delta^L_{mk}}$. Combining the above decomposition with (20), we obtain (48).

B.2 Lemma 1 with Sectoral Input-Output Linkages

Assume trade is frictionless both within and across countries. For analytical convenience, we impose that input–output coefficients are identical across regions worldwide within each sector, i.e., $\mathbf{\Phi}_m \equiv \mathbf{\Phi}$. The market-clearing condition for sector j in region m is given by

$$W_{mj}(s)L_{mj} = \delta^L_j \frac{(W_{mj}(s)^{\delta^L_j}/A_{mj}(s))^{-\theta_j}}{\sum_{m'}(W_{m'j}(s)^{\delta^L_j}/A_{m'j}(s))^{-\theta_j}}\zeta_j. \tag{B.11}$$

Here, $\zeta_j = \sum_k \zeta^k_j\alpha_k$, with ζ^k_j the $\{j,k\}$-element of $[\mathbf{I}-\mathbf{\Phi}]^{-1}$. Intermediate input costs cancel out because, under frictionless trade, they are equalized across regions and sectors.

We can solve for wages analytically,

$$W_{mj}(s) = \delta_j^L \zeta_j \frac{A_{mj}(s)^{\frac{\theta_j}{\theta_j \delta_j^L + 1}} L_{mj}^{-\frac{1}{\theta_j \delta_j^L + 1}}}{\sum_{m'} A_{m'j}(s)^{\frac{\theta_j}{\theta_j \delta_j^L + 1}} L_{m'j}^{\frac{\theta_j \delta_j^L}{\theta_j \delta_j^L + 1}}}, \tag{B.12}$$

as well as trade shares,

$$\pi_{mj}(s) = \frac{A_{mj}(s)^{\frac{\theta_j}{\theta_j \delta_j^L + 1}} L_{mj}^{\frac{\theta_j \delta_j^L}{\theta_j \delta_j^L + 1}}}{\sum_{m'} A_{m'j}(s)^{\frac{\theta_j}{\theta_j \delta_j^L + 1}} L_{m'j}^{\frac{\theta_j \delta_j^L}{\theta_j \delta_j^L + 1}}}$$

Using (6), we can further solve for the final-good price index

$$P_{mj}(s) = C_j \left(\sum_{m' \in M} A_{m'j}(s)^{\frac{\theta_j}{\theta_j \delta_j^L + 1}} L_{m'j}^{\frac{\theta_j \delta_j^L}{\theta_j \delta_j^L + 1}} \right)^{-\frac{1 + \theta_j \delta_j^L}{\theta_j}} \prod_k P_{mk}^{\delta_j^k},$$

where C_j is an industry-specific constant. Taking logs yields

$$\log P_{mj}(s) = \sum_k \zeta_j^k \left(\log C_k - \frac{1 + \theta_k \delta_k^L}{\theta_k} \log \left(\sum_{m' \in M} A_{m'k}(s)^{\frac{\theta_k}{\theta_k \delta_k^L + 1}} L_{m'k}^{\frac{\theta_k \delta_k^L}{\theta_k \delta_k^L + 1}} \right) \right)$$

where ζ_j^k is the (j,k) element of $[\mathbf{I} - \mathbf{\Phi}]^{-1}$.

Taking logs on (B.12) yields

$$\log W_{mj}(s) = \log \delta_j^L \zeta_j + \frac{\theta_j}{\theta_j \delta_j^L + 1} \log A_{mj}(s) - \frac{1}{\theta_j \delta_j^L + 1} \log L_{mj} - \log \sum_{m'} A_{m'j}(s)^{\frac{\theta_j}{\theta_j \delta_j^L + 1}} L_{m'j}^{\frac{\theta_j \delta_j^L}{\theta_j \delta_j^L + 1}}.$$

The final-good price is given by

$$\log P_m(s) = \sum_j \sum_k \alpha_j \zeta_j^k \left(\log C_k - \frac{1 + \theta_k \delta_k^L}{\theta_k} \log \left(\sum_{m' \in M} A_{m'k}(s)^{\frac{\theta_k}{\theta_k \delta_k^L + 1}} L_{m'k}^{\frac{\theta_k \delta_k^L}{\theta_k \delta_k^L + 1}} \right) \right).$$

We calculate fluctuations of wages and prices around the deterministic equilibrium by linearizing the two previous expressions,

$$\hat{W}_{mj}(s) = \frac{\theta_j}{\theta_j \delta_j^L + 1} \hat{A}_{mj}(s) - \sum_{m'} \bar{\pi}_{m'j} \frac{\theta_j}{\theta_j \delta_j^L + 1} \hat{A}_{m'j}(s), \tag{B.13}$$

and

$$\hat{P}_m(s) = -\sum_k \nu_k \sum_{m'} \bar{\pi}_{m'k} \hat{A}_{m'k}(s), \tag{B.14}$$

where $\nu_k \equiv \sum_j \alpha_j \zeta_j^k$, and $\hat{A}_{mj}(s) = \hat{A}_{i,j}(s)$ for all $m \in M_i$.

Hence,

$$\hat{W}_{mj}(s) - \hat{P}_m(s) = \left(\frac{\theta_j}{\theta_j \delta_j^L + 1}(1 - \bar{\pi}_{i,j}) + \nu_j \bar{\pi}_{i,j}\right) \hat{A}_{i,j}(s) - \left(\frac{\theta_j}{\theta_j \delta_j^L + 1} - \nu_j\right) \sum_{i' \neq i} \bar{\pi}_{i',j} \hat{A}_{i',j}(s)$$
$$+ \sum_{k \neq j} \sum_{i'} \nu_k \bar{\pi}_{i',k} \hat{A}_{i',k}(s).$$

where $\bar{\pi}_{i,j} \equiv \sum_{m \in M_i} \bar{\pi}_{mj}$. With i.i.d. shocks, $\Sigma_{ij,i'k} = 0$ for all $i \neq i'$ and $j \neq k$ so that

$$\Xi_{i,j} = \left(\frac{\theta_j}{\theta_j \delta_j^L + 1}(1 - \bar{\pi}_{i,j}) + \nu_j \bar{\pi}_{i,j}\right)^2 \sigma_{i,j}^2 + \left(\frac{\theta_j}{\theta_j \delta_j^L + 1} - \nu_j\right)^2 \sum_{i' \neq i} (\bar{\pi}_{i',j})^2 \sigma_{i',j}^2 + \sum_{k \neq j} \sum_{i'} \nu_k^2 (\bar{\pi}_{i',k})^2 \sigma_{i',k}^2.$$

Under autarky, $\bar{\pi}_{i,j} = 1$ and $\bar{\pi}_{i',j} = 0$ for all $i' \neq i$. Then, the first term becomes $\nu_j^2 \sigma_{i,j}^2$ and all the cross-country terms vanish,

$$\Xi_{i,j}^A = \sum_j \nu_k^2 \sigma_{i,k}^2 = \Xi_i^A.$$

B.3 Proposition 2 with Sectoral Input-Output Linkages

Assume trade is frictionless both within and across countries. We still impose that input–output coefficients are identical across regions worldwide within each sector, i.e., $\mathbf{\Phi}_m \equiv \mathbf{\Phi}$. From (18),

$$\mathbb{U}_i^{1/(1-\gamma)} = \Gamma\left(\frac{\kappa + \gamma - 1}{\kappa}\right)^{\frac{1}{1-\gamma}} \left(\sum_j U_{i,j}^{\frac{\kappa}{1-\gamma}}\right)^{\frac{1}{\kappa}}.$$

Applying the second-order approximation

$$U_{i,j} \approx \frac{1}{1-\gamma} \left(\frac{\bar{W}_{i,j}}{\bar{P}_i}\right)^{1-\gamma} \left(1 - \frac{\gamma(1-\gamma)}{2} \Xi_{i,j}\right),$$

we obtain

$$\mathbb{U}_i^{\frac{1}{1-\gamma}} \approx C \left(\sum_j \left(\frac{\bar{W}_{i,j}}{\bar{P}_i}\right)^{\kappa} \left(1 - \frac{\gamma(1-\gamma)}{2} \Xi_{i,j}\right)^{\frac{\kappa}{1-\gamma}}\right)^{\frac{1}{\kappa}},$$

for a constant $C > 0$. Using the expression for labor shares,

$$\lambda_{i,j} = \frac{(\bar{W}_{i,j}/\bar{P}_i)^{\kappa}\left(1 - \frac{\gamma(1-\gamma)}{2}\Xi_{i,j}\right)^{\frac{\kappa}{1-\gamma}}}{\sum_{j'}(\bar{W}_{i,j'}/\bar{P}_i)^{\kappa}\left(1 - \frac{\gamma(1-\gamma)}{2}\Xi_{i,j'}\right)^{\frac{\kappa}{1-\gamma}}},$$

it follows that

$$\mathbb{U}_i^{\frac{1}{1-\gamma}} \approx C_1 \prod_k \left(1 - \frac{\gamma(1-\gamma)}{2}\Xi_{i,k}\right)^{\frac{\sum_j \alpha_j \zeta_j^k \delta_k^L}{1-\gamma}} \prod_k \lambda_{i,k}^{-\frac{\sum_j \alpha_j \zeta_j^k \delta_k^L}{\kappa}} \frac{\prod_k \bar{W}_{i,k}^{\sum_j \alpha_j \zeta_j^k \delta_k^L}}{\bar{P}_i}. \tag{B.15}$$

Next, using

$$\frac{\bar{P}_{ij}^A}{\bar{P}_{ij}} = \prod_k (\bar{\pi}_{ii}^k)^{-\frac{\zeta_j^k}{\theta_k}} \prod_k \left(\frac{\lambda_{ik}}{\lambda_{ik}^A}\right)^{\frac{\kappa-1}{\kappa}\zeta_j^k \rho_k} \prod_k \left(\frac{\bar{W}_{ik}^A}{\bar{W}_{ik}}\right)^{\zeta_j^k \delta_k^L},$$

yields

$$\begin{aligned} GT_i = &\prod_k \left(\frac{\lambda_{i,k}}{\lambda_{i,k}^A}\right)^{-\frac{\sum_j \alpha_j \zeta_j^k \delta_k^L}{\kappa}} \times \prod_k \left(\frac{1 - \frac{\gamma(1-\gamma)}{2}\Xi_{i,k}}{1 - \frac{\gamma(1-\gamma)}{2}\Xi_{i,k}^A}\right)^{\frac{\sum_j \alpha_j \zeta_j^k \delta_k^L}{1-\gamma}} \times \prod_k \left(\bar{\pi}_{i,i}^k\right)^{-\sum_j \frac{\alpha_j \zeta_j^k}{\theta_k}} \\ &\times \prod_k \left(\frac{\lambda_{i,k}}{\lambda_{i,k}^A}\right)^{\sum_j \alpha_j \zeta_j^k \rho_k} \times \prod_k \left(\frac{\lambda_{i,k}}{\lambda_{i,k}^A}\right)^{-\frac{1}{\kappa}\sum_j \alpha_j \zeta_j^k \rho_k}. \end{aligned} \tag{B.16}$$

In autarky, $\lambda_{i,j}^A = \delta_j^L \zeta_j$.

B.4 Derivation of Equation (48) with AD securities.

We obtain the result for the general partial insurance case, which nests the full insurance case ($\mu_i = 1$) and the financial autarky case ($\mu_i = 0$). Following the proof in Appendix B.1, we obtain:

$$\frac{\bar{W}_{mj}/\bar{P}_m}{\bar{W}_{mj}^A/\bar{P}_m^A} = \underbrace{\frac{\bar{\omega}_{mj}}{\bar{\omega}_{mj}^A}}_{\text{Workers Heterogeneity}} \times \underbrace{\prod_k \left(\frac{\bar{\pi}_{mm}^k}{\bar{\pi}_{mm}^{k,A}}\right)^{-\sum_j \frac{\alpha_j \zeta_{mj}^k}{\theta_k}}}_{\text{ACR}} \times \underbrace{\prod_k \left(\frac{\mu_i(\lambda_{mk}^{AD})^{1-1/\kappa} + (1-\mu_i)(\lambda_{mk}^H)^{1-1/\kappa}}{\mu_i(\lambda_{mk}^{AD,A})^{1-1/\kappa} + (1-\mu_i)(\lambda_{mk}^{H,A})^{1-1/\kappa}}\right)^{\sum_j \alpha_j \zeta_{mj}^k \rho_k}}_{\text{Economies of scale + interaction}} \tag{B.17}$$

where the difference comes from incorporating that the employment changes depend on both workers with access to AD securities and workers without access to AD securities.

We thus obtain:

$$\left(\frac{U_{mj}^{AD}}{U_{mj}^{AD,A}}\right)^{\frac{1}{1-\gamma}} \approx \underbrace{\left(\frac{\bar{\omega}_{mj}}{\bar{\omega}_{mj}^{A}}\right)}_{\text{Worker heterogeneity}} \times \underbrace{\prod_k \left(\frac{\bar{\pi}_{mm}^{k}}{\bar{\pi}_{mm}^{k,A}}\right)^{-\sum_j \frac{\alpha_j \zeta_{mj}^{k}}{\theta_k}}}_{\text{ACR}} \times \underbrace{\prod_k \left(\frac{\mu_i(\lambda_{mk}^{AD})^{1-1/\kappa} + (1-\mu_i)(\lambda_{mk}^{H})^{1-1/\kappa}}{\mu_i(\lambda_{mk}^{AD,A})^{1-1/\kappa} + (1-\mu_i)(\lambda_{mk}^{H,A})^{1-1/\kappa}}\right)^{\sum_j \alpha_j \zeta_{mj}^{k} \rho_k}}_{\text{Economies of scale + interaction}}$$

$$\times \underbrace{\left(\frac{1 - \frac{(1-\gamma)\gamma}{2}\Xi_{m,j}^{AD}}{1 - \frac{(1-\gamma)\gamma}{2}\Xi_{m,j}^{AD,A}}\right)^{\frac{1}{1-\gamma}}}_{\text{Volatility of real wage}}, \tag{B.18}$$

$$\left(\frac{U_{mj}^{H}}{U_{mj}^{H,A}}\right)^{\frac{1}{1-\gamma}} \approx \underbrace{\left(\frac{\bar{\omega}_{mj}}{\bar{\omega}_{mj}^{A}}\right)}_{\text{Worker heterogeneity}} \times \underbrace{\prod_k \left(\frac{\bar{\pi}_{mm}^{k}}{\bar{\pi}_{mm}^{k,A}}\right)^{-\sum_j \frac{\alpha_j \zeta_{mj}^{k}}{\theta_k}}}_{\text{ACR}} \times \underbrace{\prod_k \left(\frac{\mu_i(\lambda_{mk}^{AD})^{1-1/\kappa} + (1-\mu_i)(\lambda_{mk}^{H})^{1-1/\kappa}}{\mu_i(\lambda_{mk}^{AD,A})^{1-1/\kappa} + (1-\mu_i)(\lambda_{mk}^{H,A})^{1-1/\kappa}}\right)^{\sum_j \alpha_j \zeta_{mj}^{k} \rho_k}}_{\text{Economies of scale + interaction}}$$

$$\times \underbrace{\left(\frac{1 - \frac{(1-\gamma)\gamma}{2}\Xi_{m,j}^{H}}{1 - \frac{(1-\gamma)\gamma}{2}\Xi_{m,j}^{H,A}}\right)^{\frac{1}{1-\gamma}}}_{\text{Volatility of real wage}}, \tag{B.19}$$

separately for workers with access to AD securities and those without.

B.5 Lemma 2 with Sectoral Input-Output Linkages

In the case of global AD securities with $\mu_i = \mu$ for all i, according to equation (B.14), we derive:

$$\Xi_{i,j}^{AD} = \text{var}(\hat{P}_i(s)) = \sum_k \sum_{i'} \nu_k^2 (\bar{\pi}_{i',k})^2 \sigma_{i',k}^2 \tag{B.20}$$

In the case of country-level AD securities, we note that:

$$\hat{Y}_i^{AD}(s) = \hat{Y}_i(s) = \sum_k s_{ik} \hat{W}_{ik} = \sum_k s_{ik} \left[\frac{\theta_k}{\theta_k \delta_k^L + 1} \hat{A}_{mk}(s) - \sum_{m'} \bar{\pi}_{m'k} \frac{\theta_k}{\theta_k \delta_k^L + 1} \hat{A}_{m'k}(s)\right] \tag{B.21}$$

where the expression of $\hat{W}_{ij}$ follows equation (B.13). Combining it with (B.14), we obtain:

$$\Xi_{i,j}^{AD} = \text{var}(\hat{Y}_i^{AD}(s) - \hat{P}_i(s)) = \sum_k \left[\left(\frac{\theta_k}{1 + \delta_k^L \theta_k} s_{ik}(1 - \bar{\pi}_{i,k}) + \nu_k \bar{\pi}_{i,k}\right)^2 \sigma_{i,k}^2 + \left(s_{ik} \frac{\theta_k}{1 + \delta_k^L \theta_k} - \nu_k\right)^2 \sum_{i' \neq i} (\bar{\pi}_{i',k})^2 \sigma_{i',k}^2\right] \tag{B.22}$$

B.6 Proposition 4 with Sectoral Input-Output Tables

Following the proof in Appendix B.3, we obtain the utility change separately for workers with access to AD securities and without access:

$$\left(\frac{U_i^{AD}}{U_i^{AD,A}}\right)^{\frac{1}{1-\gamma}} = \prod_k \left(\frac{\lambda_{i,k}^{AD}}{\lambda_{i,k}^{AD,A}}\right)^{-\frac{\sum_j \alpha_j \zeta_j^k \delta_k^L}{\kappa}} \times \prod_k \left(\frac{1-\frac{\gamma(1-\gamma)}{2}\Xi_{i,k}^{AD}}{1-\frac{\gamma(1-\gamma)}{2}\Xi_{i,k}^{AD,A}}\right)^{\frac{\sum_j \alpha_j \zeta_j^k \delta_k^L}{1-\gamma}} \times \prod_k \left(\bar{\pi}_{i,i}^k\right)^{-\sum_j \frac{\alpha_j \zeta_j^k}{\theta_k}} \times \prod_k \left(\frac{\mu_i(\lambda_{mk}^{AD})^{1-1/\kappa} + (1-\mu_i)(\lambda_{mk}^{H})^{1-1/\kappa}}{\mu_i(\lambda_{mk}^{AD,A})^{1-1/\kappa} + (1-\mu_i)(\lambda_{mk}^{H,A})^{1-1/\kappa}}\right)^{\sum_j \alpha_j \zeta_j^k \rho_k} \tag{B.23}$$

$$\left(\frac{U_i^{H}}{U_i^{H,A}}\right)^{\frac{1}{1-\gamma}} = \prod_k \left(\frac{\lambda_{i,k}^{H}}{\lambda_{i,k}^{H,A}}\right)^{-\frac{\sum_j \alpha_j \zeta_j^k \delta_k^L}{\kappa}} \times \prod_k \left(\frac{1-\frac{\gamma(1-\gamma)}{2}\Xi_{i,k}^{H}}{1-\frac{\gamma(1-\gamma)}{2}\Xi_{i,k}^{H,A}}\right)^{\frac{\sum_j \alpha_j \zeta_j^k \delta_k^L}{1-\gamma}} \times \prod_k \left(\bar{\pi}_{i,i}^k\right)^{-\sum_j \frac{\alpha_j \zeta_j^k}{\theta_k}} \times \prod_k \left(\frac{\mu_i(\lambda_{mk}^{AD})^{1-1/\kappa} + (1-\mu_i)(\lambda_{mk}^{H})^{1-1/\kappa}}{\mu_i(\lambda_{mk}^{AD,A})^{1-1/\kappa} + (1-\mu_i)(\lambda_{mk}^{H,A})^{1-1/\kappa}}\right)^{\sum_j \alpha_j \zeta_j^k \rho_k} \tag{B.24}$$

where the difference comes from incorporating that the employment changes depend on both workers with access to AD securities and workers without access to AD securities.

Given that $\mathbb{U}_i(\mu_i) \equiv \mu_i \mathbb{U}_i^{AD} + (1-\mu_i)\mathbb{U}_i^{H}$ and that the autarkic equilibrium is the same for both types, we obtain the gains from trade as:

$$GT_i(\mu_i) \approx \prod_k \left(\bar{\pi}_{i,i}^k\right)^{-\sum_j \frac{\alpha_j \zeta_j^k}{\theta_k}} \times \prod_k \left(\frac{\mu_i(\lambda_{mk}^{AD})^{1-1/\kappa} + (1-\mu_i)(\lambda_{mk}^{H})^{1-1/\kappa}}{\mu_i(\lambda_{mk}^{AD,A})^{1-1/\kappa} + (1-\mu_i)(\lambda_{mk}^{H,A})^{1-1/\kappa}}\right)^{\sum_j \alpha_j \zeta_j^k \rho_k} \left[\mu_i \prod_k \left(\frac{\lambda_{i,k}^{AD}}{\lambda_{i,k}^{AD,A}}\right)^{-\frac{(1-\gamma)\sum_j \alpha_j \zeta_j^k \delta_k^L}{\kappa}} \times \prod_k \left(\frac{1-\frac{\gamma(1-\gamma)}{2}\Xi_{i,k}^{AD}}{1-\frac{\gamma(1-\gamma)}{2}\Xi_{i,k}^{AD,A}}\right)^{\sum_j \alpha_j \zeta_j^k \delta_k^L} + (1-\mu_i)\prod_k \left(\frac{\lambda_{i,k}^{H}}{\lambda_{i,k}^{H,A}}\right)^{-\frac{(1-\gamma)\sum_j \alpha_j \zeta_j^k \delta_k^L}{\kappa}} \times \prod_k \left(\frac{1-\frac{\gamma(1-\gamma)}{2}\Xi_{i,k}^{H}}{1-\frac{\gamma(1-\gamma)}{2}\Xi_{i,k}^{H,A}}\right)^{\sum_j \alpha_j \zeta_j^k \delta_k^L}\right]^{\frac{1}{1-\gamma}}. \tag{B.25}$$

C Data

C.1 Countries and Sectors

We consider the following countries: Austria; Belgium; Canada; China; Czech Republic; Germany; Denmark; Spain; Estonia; Finland; France; the United Kingdom; Greece; Hungary; Italy; Japan; Lithuania; Latvia; Mexico; Netherlands; Norway; Poland; Portugal; Slovakia; Slovenia; and the United States. We also construct a Rest of the World aggregating all the other countries.

Table C.1 presents the 20 aggregated sectors we consider in the calibration, including agriculture, mining, 17 manufacturing sectors (considered in Section 5), and services. The industry classification is based on the International Standard Industrial Classification (ISIC)

Table C.1: Industry Classification Based on ISIC Revision 4.

Industry Code	Description	$\frac{\kappa-1}{\kappa}\rho_j$	θ_j
D01T03	Agriculture	0	4.5
D05T09	Mining	0	4.5
D10T12	Manuf. of food products, beverages, and tobacco products	0.24	3.6
D13T15	Manuf. of textiles, wearing apparel, leather and related products	0.14	8.1
D16	Manuf. of wood	0.15	5.9
D17T18	Manuf. of paper and paper products; printing and reproduction of recorded media	0.18	5.8
D19	Manuf. of coke and refined petroleum products	0.09	9
D20	Manuf. of chemicals and chemical products	0.27	3.1
D21	Manuf. of basic pharmaceutical products	0.27	3.1
D22	Manuf. of rubber and plastics products	0.45	1.7
D23	Manuf. of other non-metallic mineral products	0.19	5.1
D24	Manuf. of basic metals	0.11	8.9
D25	Manuf. of fabricated metal products, except machinery and equipment	0.14	7
D26	Manuf. of computer, electronic and optical products	0.09	10.8
D27	Manuf. of electrical equipment	0.10	10.8
D28	Manuf. of machinery and equipment n.e.c.	0.28	3.3
D29	Manuf. of motor vehicles, trailers and semi-trailers	0.20	4.5
D30	Manuf. of other transport equipment	0.20	4.5
D31T33	Manuf. of furniture and other manufacturing, repair	0.20	4.5
D35T99	Services	0	4.5

Note: Estimates of $\frac{\kappa-1}{\kappa}\rho_j$ and θ_j from Bartelme, Costinot, Donaldson and Rodríguez-Clare (2025).

Revision 4.

C.2 Constructing Bilateral Trade Flows For Subnational Geographies

We use the Commodity Flow Survey (CFS) for 2017 to construct subnational bilateral trade flows for CZ's across the United States. One issue is that the trade flows in the survey are not based on the finer geographic levels we consider (CZs). We adopt a similar procedure in the literature (Allen and Arkolakis 2014, Monte et al. 2018, Fajgelbaum and Gaubert 2020) to deal with this issue as follows.

First, for each tradable sector j, we parameterize the trade costs between subnational regions in the United States as $\log \tau_{lm}^j = \zeta_j \log dist_{lm} + e_{lm}$, where $dist_{lm}$ is the distance between region l and region m. We follow Allen and Arkolakis (2014) to estimate ζ_j using trade flows between CFS areas, controlling for origin and destination fixed effects. We then set trade costs between CZs to $\tau_{lm}^j = dist_{lm}^{\zeta_j}$ for any two different CZs $l, m \in M_{US}$. The intra-region trade costs are $\tau_{ll}^j = 1 \; \forall \; l \in M_{US}$.

Second, the good market clearing for region l and sector j can be written as

$$Y_{lj}^{US} = \sum_{m \in M_{US}} \frac{D_{lj}(\tau_{lm}^j)^{-\theta_j}}{\sum_{l' \in M_{US}} D_{l'j}(\tau_{l'm}^j)^{-\theta_j}} X_{mj}^{US}, \tag{C.1}$$

where D_{lj} is a region-industry-specific fixed effect, capturing region-industry-specific productivity and production costs. Y_{lj}^{US} is the production value in region l and sector j, and X_{mj}^{US} is the expenditures on goods from sector j in m. We use superscript US to denote that these values are net of exports and imports. Production and expenditure values are constructed from the County Business Patterns.[20] Along with the trade costs τ_{lm}^{j} constructed from the first step, we can use (C.1) to solve for D_{lj} and the implied trade flows between CZs.

D Empirical Evidence: Cross-Country Results

We draw sectoral output, value-added, and employment for each country from the OECD STAN Database. The data spans from 1970 and 2019 with shorter periods (at least 20 years) for some countries. Because the data for several countries end in 2017 and spans fewer years than for the United States, we focus on the impact of uncertainty on employment changes between 1977–2017. We concentrate on the same 17 manufacturing sectors featured in our empirical analysis.

To obtain a measure of uncertainty, we first compute growth in average real value-added per worker by country, industry, and year,

$$growth_{jt} = \log\left(\frac{VA_{i,j,t}}{Emp_{i,j,t}}\right) - \log\left(\frac{VA_{i,j,t-1}}{Emp_{i,j,t-1}}\right).$$

We then compute the change in uncertainty between 1977-1997 and 1997-2017 for country i and industry j based on changes in the variance of productivity growth,

$$\Delta v_{i,j} = var(\underbrace{growth_{i,j,t}}_{t=1997,\ldots,2017}) - var(\underbrace{growth_{i,j,t}}_{t=1977,\ldots,1997}).$$

We estimate the following equation:

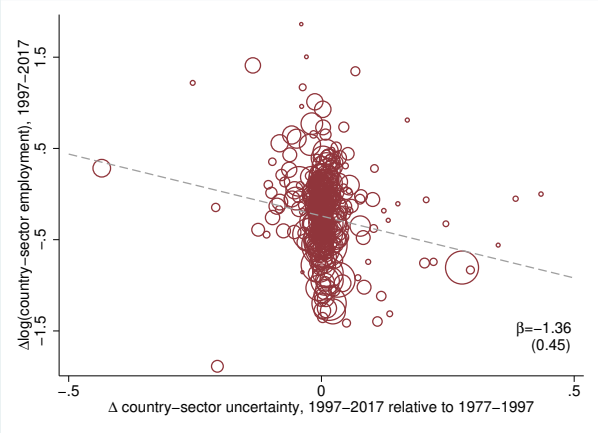
$$\Delta \log(L_{i,j}) = \beta_0 + \beta_1 \Delta v_{i,j} + \beta_2 \Delta v_{i,j} \times \tilde{\rho}_j + \beta_3 X_{i,j} + \epsilon_{i,j}.$$

$\Delta \log(L_{ij})$ represents changes in industry employment for country i between 1997 and 2017, $\tilde{\rho}_j$ is the scale parameter for industry j, from Bartelme, Costinot, Donaldson and Rodríguez-Clare (2025), and $X_{i,j}$ are other country-industry controls. In some specifications, we include country fixed effects and industry fixed effects.

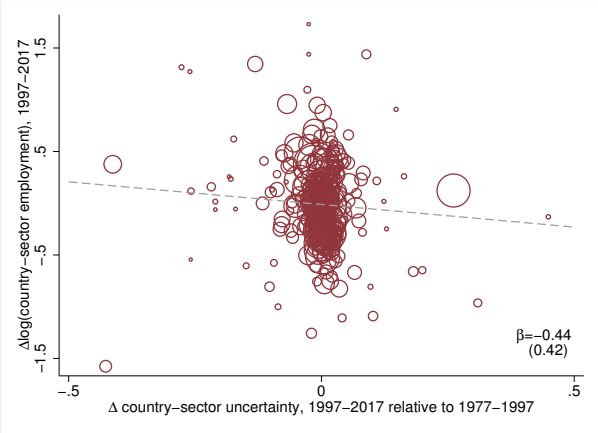
[20] With the production data, we can use input-output tables to construct intermediate-input expenditures on goods sourced from each sector. Then we can use labor income and workers' expenditure shares to construct final-good expenditures on goods sourced from each sector.

Figure D.1: Country-sector Employment Growth and Uncertainty.

(a) Raw data



(b) Control by global industry trend



Note: The circles are proportional to the region employment level in 1997. In Panel (b), we first demean country-sector-level employment and uncertainty changes by the global averages.

E Calibration of Correlation Matrix of Productivity Shocks

Shocks $A_{i,j}(s)$ are drawn from a log-normal distribution. Since countries have data time series of unequal length, the empirically computed correlation matrix is not guaranteed to be positive definite. To address this issue, we model shocks using a multi-factor structure with year-specific global, country, and sector components. This approach accommodates an unbalanced panel and guarantees a positive-definite covariance structure by construction.

Specifically, we estimate a fully flexible factor model in which the loadings on the global, country, and sector factors are country-specific, and the residual standard deviations are country–sector specific,

$$g_{i,j,t} = \beta_i^G f_t + \beta_i^C f_{i,t} + \beta_i^S f_{j,t} + \sigma_{i,j}^{\eta} \eta_{i,j,t}, \qquad \eta_{i,j,t} \sim \mathcal{N}(0,1).$$

Here, f_t is a standardized global factor, $f_{i,t}$ is a standardized country factor, and $f_{j,t}$ is a standardized sector factor. We assume that

$$f_t \sim \mathcal{N}(0,1), \quad f_{i,t} \sim \mathcal{N}(0,1), \quad f_{j,t} \sim \mathcal{N}(0,1),$$

and that all factors are mutually independent. The idiosyncratic shocks $\eta_{i,j,t}$ are independently distributed across countries and sectors.

After estimating β_i^G, β_i^C, β_i^S, and $\sigma_{i,j}^{\eta}$ using data on country-sector-specific growth of value added per worker, the implied correlation between (i,j) and (i',j') is

$$\hat{r}_{(i,j),(i',j')} = \frac{\hat{\beta}_i^G \hat{\beta}_{i'}^G + \hat{\beta}_i^C \hat{\beta}_{i'}^C \mathbf{1}\{i=i'\} + \hat{\beta}_i^S \hat{\beta}_{i'}^S \mathbf{1}\{j=j'\} + (\hat{\sigma}_{i,j}^{\eta})^2 \mathbf{1}\{i=i', j=j'\}}{\sqrt{(\hat{\beta}_i^G)^2 + (\hat{\beta}_i^C)^2 + (\hat{\beta}_i^S)^2 + (\hat{\sigma}_{i,j}^{\eta})^2} \sqrt{(\hat{\beta}_{i'}^G)^2 + (\hat{\beta}_{i'}^C)^2 + (\hat{\beta}_{i'}^S)^2 + (\hat{\sigma}_{i',j'}^{\eta})^2}}.$$

Table D.1: Changes in Employment and Uncertainty, by Country and Industry.

	Δlog(country-industry employment), 97–17			
	(1)	(2)	(3)	(4)
$\Delta v_{i,j}$, 97–17 relative to 77–97	-1.358*** (0.397)	-0.413 (0.344)	-0.485* (0.289)	0.228 (1.887)
$\Delta v_{i,j} \times \tilde{\rho}_j$				-0.755 (2.026)
Controls	no	no	yes	yes
Country FE	no	yes	yes	yes
Sector FE	no	yes	yes	yes
Obs	373	373	373	373
R-squared	0.039	0.583	0.595	0.596

Note: Regressions are weighted by initial sectoral employment shares within each country. Controls include productivity growth and the China shock between 1997 and 2017. Standard errors are clustered by country. * 10% ** 5% *** 1%.

Let $\mathbf{C}$ denote the $NJ \times NJ$ correlation matrix whose $((i-1)J+j, (i'-1)J+j')$ element equals $\hat{r}_{(i,j),(i',j')}$. Let L_C be the Cholesky decomposition of $\mathbf{C}$.

The productivity shocks in the simulations are then generated as

$$\mathbf{A}(s) = \mathbf{\Sigma}\, L_C\, \boldsymbol{\epsilon}(s),$$

where $\mathbf{A}(s)$ is an $NJ \times 1$ vector whose $((i-1)J+j)$-th element is $\log A_{i,j}(s)$. The matrix $\mathbf{\Sigma}$ is diagonal with $((i-1)J+j, (i-1)J+j)$ element equal to $\sigma_{i,j}$, which we estimate in the quantitative analysis. The vector $\boldsymbol{\epsilon}(s)$ is an $NJ \times 1$ vector of i.i.d. standard normal shocks. Finally, we normalize $\log A_{i,j}(s)$ by subtracting $\frac{\sigma_{i,j}^2}{2}$ so that $A_{i,j}(s)$ has unit expectation. The variance-covariance matrix of log productivity shocks is therefore $\mathbf{R} = \mathbf{\Sigma C \Sigma}$.

F Variance Decomposition

Note that:

$$P_{lj,i}(s) = C_j \left(\sum_{i'} \sum_{m \in M_{i'}} \left(\tau_{ml}^{j} \frac{c_{mj}(s)}{A_{i',j}(s)} \right)^{-\theta_j} \tilde{C}_{mj} \lambda_{mj}^{\tilde{\rho}_j} L_{i'}^{\rho_j \theta_j} \right)^{-\frac{1}{\theta_j}}, \tag{F.1}$$

where

$$c_{mj}(s) = \left(\frac{W_{mj}(s)}{\delta_{mj}^{L}} \right)^{\delta_{mj}^{L}} \prod_{k} \left(\frac{P_{mk}(s)}{\delta_{mj}^{k}} \right)^{\delta_{mj}^{k}}. \tag{F.2}$$

Thus, by lineralization, we have

$$\hat{P}_{lj}(s) = \sum_m \bar{\pi}^j_{ml,i} \left[\delta^L_{mj} \hat{W}_{mj}(s) + \sum_k \delta^k_{mj} \hat{P}_{mk}(s) - \hat{A}_{i',j}(s) \right] \tag{F.3}$$

Define $\hat{\mathbf{P}} = [\hat{P}_{lj}(s)]$ as the $(N \times J, 1)$ vector, where the $((l-1) \times J + j, 1)$ element is $\hat{P}_{lj}(s)$. We define $\hat{\mathbf{W}} = [\hat{W}_{lj}(s)]$ as the $(N \times J, 1)$ vector, where the $((l-1) \times J + j, 1)$ element is $\hat{W}_{lj}(s)$. We define $\hat{\mathbf{A}} = [\hat{A}_{i,j}(s)]$ as the $(N \times J, 1)$ vector, where the $((l-1) \times J + j, 1)$ element is $\hat{A}_{i,j}(s)$, where i is the country l belongs to.

Define $\mathbf{\Pi} = [\bar{\pi}^j_{ml,i}]$ as the $(M \times J, M \times J)$ matrix, where the $((l-1) \times J + j, (m-1) \times J + j)$ element is $\bar{\pi}^j_{ml,i}$ and zero otherwise. Define $\mathbf{\Delta}^L = \text{diag}(\delta^L_{lj})$, which is the diagonal matrix with the $((l-1) \times J + j, (l-1) \times J + j)$ as δ^L_{lj}. Define $\mathbf{\Delta}^I = (\delta^k_{lj})$ as the $(M \times J, M \times J)$ matrix, where the $((l-1) \times J + j, (l-1) \times J + k)$ element is δ^k_{lj} and zero otherwise. We can recast the above equation in matrix form:

$$\hat{\mathbf{P}} = \mathbf{\Pi} \left[\mathbf{\Delta}^L \hat{\mathbf{W}} + \mathbf{\Delta}^I \hat{\mathbf{P}} - \hat{\mathbf{A}} \right] \tag{F.4}$$

which leads to

$$\hat{\mathbf{P}} = \left[\mathbf{I} - \mathbf{\Pi} \mathbf{\Delta}^I \right]^{-1} \mathbf{\Pi} \left[\mathbf{\Delta}^L \hat{\mathbf{W}} - \hat{\mathbf{A}} \right]. \tag{F.5}$$

We next use the market equilibrium condition for each industry in each region. Note that:

$$Y_{mj}(s) = \sum_l \pi^j_{ml}(s) E_{lj}(s) = \sum_l \pi^j_{ml}(s) \left[\sum_k \delta^j_{lk} Y_{lk}(s) + C_{lj}(s) \right] \tag{F.6}$$

where the first part in the bracket denotes intermediate input costs, and the second part denotes consumption, with $C_{lj}(s) = \alpha_j \sum_k \delta^L_{lk} Y_{lk}(s)$ in the case of no insurance.

In the case of no insurance, define $\hat{\mathbf{Y}} = [\hat{Y}_{mj}(s)]$ as the vector, where the $((m-1) \times J + j, 1)$ element is $\hat{Y}_{mj}(s)$. Define $\mathbf{S} = [\bar{s}^j_{ml}]$ as the $(M \times J, M \times J)$ matrix, where the $((m-1) \times J + j, (l-1) \times J + j)$ element is $\bar{s}^j_{ml} = \frac{\bar{\pi}^j_{ml} \left[\sum_k \delta^j_{lk} \bar{Y}_{lk} + \bar{C}_{lj} \right]}{\bar{Y}_{mj}}$ and zero otherwise. In the case of no insurance, define $\mathbf{\Delta}^Y$ as the $(M \times J, M \times J)$ matrix, where the $((l-1) \times J + j, (l-1) \times J + k)$ element is $\frac{\delta^j_{lk} \bar{Y}_{lk} + \alpha_j \delta^L_{lk} \bar{Y}_{lk}}{\sum_k (\delta^j_{lk} + \alpha_j \delta^L_{lk}) \bar{Y}_{lk}}$ and zero otherwise.

Define $\hat{\mathbf{\Pi}}$ as the matrix containing proportional change in trade shares, which equal:

$$\hat{\mathbf{\Pi}} = -\mathbf{\Theta} \left[\mathbf{\Delta}^L \hat{\mathbf{W}} + \mathbf{\Delta}^I \hat{\mathbf{P}} - \hat{\mathbf{A}} \right] + \mathbf{\Theta} \mathbf{S} \hat{\mathbf{P}} \tag{F.7}$$

where $\mathbf{\Theta}$ is a diagonal matrix with $((m-1) \times J + j, (m-1) \times J + j)$ element is θ_j.

Thus, we have:

$$\hat{\mathbf{Y}} = \mathbf{S} \mathbf{\Delta}^Y \hat{\mathbf{Y}} + \hat{\mathbf{\Pi}} = \mathbf{S} \mathbf{\Delta}^Y \hat{\mathbf{Y}} + \left[-\mathbf{\Theta} + \left[-\mathbf{\Theta} \mathbf{\Delta}^I + \mathbf{\Theta} \mathbf{S} \right] \left[\mathbf{I} - \mathbf{\Pi} \mathbf{\Delta}^I \right]^{-1} \mathbf{\Pi} \right] \left[\mathbf{\Delta}^L \hat{\mathbf{W}} - \hat{\mathbf{A}} \right] \tag{F.8}$$

Given that labor cannot move in the presence of shocks, we have $\hat{\mathbf{Y}} = \hat{\mathbf{W}}$. Thus, we obtain:

$$\begin{aligned}\hat{\mathbf{W}} &= \left[\mathbf{I} - \mathbf{S}\boldsymbol{\Delta}^Y + \boldsymbol{\Theta}\boldsymbol{\Delta}^L + \left(\boldsymbol{\Theta}\boldsymbol{\Delta}^I - \boldsymbol{\Theta}\mathbf{S}\right)\left[\mathbf{I} - \boldsymbol{\Pi}\boldsymbol{\Delta}^I\right]^{-1}\boldsymbol{\Pi}\boldsymbol{\Delta}^L\right]^{-1}\left[\boldsymbol{\Theta} + \left(\boldsymbol{\Theta}\boldsymbol{\Delta}^I - \boldsymbol{\Theta}\mathbf{S}\right)\left[\mathbf{I} - \boldsymbol{\Pi}\boldsymbol{\Delta}^I\right]^{-1}\boldsymbol{\Pi}\right]\hat{\mathbf{A}} \\ &= \boldsymbol{\Omega}^W\hat{\mathbf{A}}\end{aligned} \tag{F.9}$$

The real wage is given by:

$$\hat{\mathbf{W}} - \boldsymbol{\Omega}^P\hat{\mathbf{P}} = \left[\mathbf{I} - \boldsymbol{\Omega}^P\left(\mathbf{I} - \boldsymbol{\Pi}\boldsymbol{\Delta}^I\right)^{-1}\boldsymbol{\Pi}\boldsymbol{\Delta}^L\right]\boldsymbol{\Omega}^W\hat{\mathbf{A}} + \boldsymbol{\Omega}^P\left(\mathbf{I} - \boldsymbol{\Pi}\boldsymbol{\Delta}^I\right)^{-1}\boldsymbol{\Pi}\hat{\mathbf{A}} = \boldsymbol{\Omega}\hat{\mathbf{A}}, \tag{F.10}$$

with

$$\boldsymbol{\Omega} \equiv \boldsymbol{\Omega}^W + \boldsymbol{\Omega}^P\left(\mathbf{I} - \boldsymbol{\Pi}\boldsymbol{\Delta}^I\right)^{-1}\boldsymbol{\Pi}\left(\mathbf{I} - \boldsymbol{\Delta}^L\boldsymbol{\Omega}^W\right). \tag{F.11}$$

where $\boldsymbol{\Omega}^P$ is the $(M \times J, M \times J)$ matrix, where the $((l-1) \times J + j, (l-1) \times J + k)$ element is α_k and zero otherwise. The first term of $\boldsymbol{\Omega}$ captures the contribution of nominal wages to real income volatility, whereas the second, $\boldsymbol{\Omega}^P\left(\mathbf{I} - \boldsymbol{\Pi}\boldsymbol{\Delta}^I\right)^{-1}\boldsymbol{\Pi}\left(\mathbf{I} - \boldsymbol{\Delta}^L\boldsymbol{\Omega}^W\right)$, represents the contribution of final-good prices. These two terms are used to decompose the variance into the relative contributions of nominal wages and final-good prices in Figure 3.

Given that the variance-covariance matrix of $\hat{\mathbf{A}}$ is $\mathbf{R} = \boldsymbol{\Sigma}\mathbf{C}\boldsymbol{\Sigma}$ (see Appendix E), we can derive the variance-covariance matrix of real wage changes as:

$$\boldsymbol{\Omega}\,\mathbf{R}\,\boldsymbol{\Omega}^T = \left[\boldsymbol{\Omega}^W + \boldsymbol{\Omega}^P\left(\mathbf{I} - \boldsymbol{\Pi}\boldsymbol{\Delta}^I\right)^{-1}\boldsymbol{\Pi}\left(\mathbf{I} - \boldsymbol{\Delta}^L\boldsymbol{\Omega}^W\right)\right]\mathbf{R}\left[\boldsymbol{\Omega}^W + \boldsymbol{\Omega}^P\left(\mathbf{I} - \boldsymbol{\Pi}\boldsymbol{\Delta}^I\right)^{-1}\boldsymbol{\Pi}\left(\mathbf{I} - \boldsymbol{\Delta}^L\boldsymbol{\Omega}^W\right)\right]^T \tag{F.12}$$

If there are no input-output linkages, i.e., $\boldsymbol{\Delta}^I = \emptyset$ and $\boldsymbol{\Delta}^L = \mathbf{I}$, $\boldsymbol{\Omega}$ reduces to $\boldsymbol{\Omega}^W + \boldsymbol{\Omega}^P\boldsymbol{\Pi}(\mathbf{I} - \boldsymbol{\Omega}^W)$ and the variance simplifies to:

$$\left[\boldsymbol{\Omega}^W + \boldsymbol{\Omega}^P\boldsymbol{\Pi}\left(\mathbf{I} - \boldsymbol{\Omega}^W\right)\right]\mathbf{R}\left[\boldsymbol{\Omega}^W + \boldsymbol{\Omega}^P\boldsymbol{\Pi}\left(\mathbf{I} - \boldsymbol{\Omega}^W\right)\right]^T. \tag{F.13}$$

where

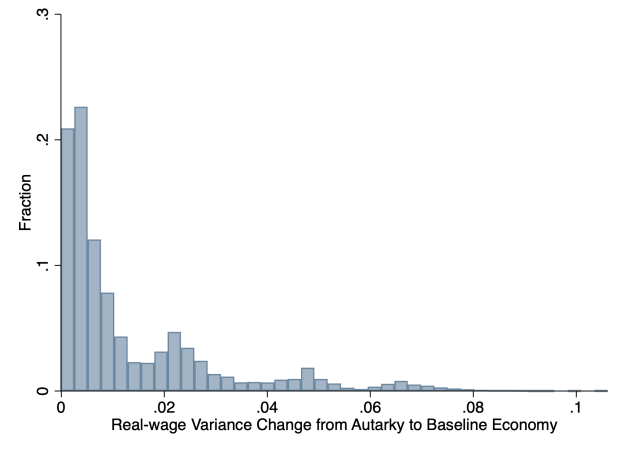
$$\boldsymbol{\Omega}^W = \left[\mathbf{I} - \mathbf{S}\boldsymbol{\Delta}^Y + \boldsymbol{\Theta}\left(\mathbf{I} - \mathbf{S}\boldsymbol{\Pi}\right)\right]^{-1}\boldsymbol{\Theta}\left(\mathbf{I} - \mathbf{S}\boldsymbol{\Pi}\right),$$

and $\boldsymbol{\Delta}^Y$ is defined by setting $\delta^j_{lk} = 0$ and $\delta^L_{lk} = 1$.

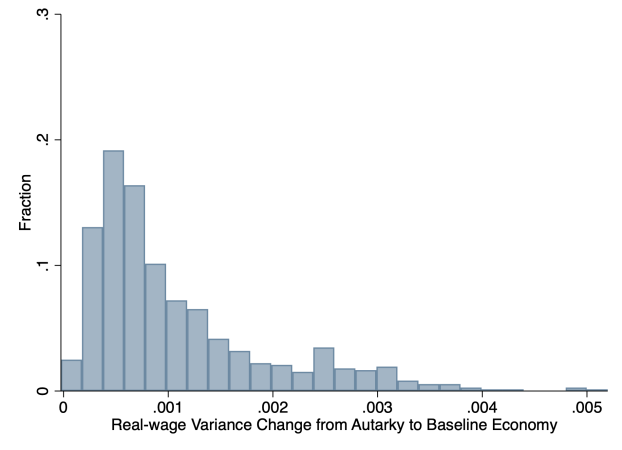
G Additional Results

Figure G.1: Real-Wage Volatility Change: Autarky to Baseline.

(a) Region-sectors



(b) Regions



Note: Change in real wage variance from autarky to the calibrated economy, in a sector-CZ pair, $\Xi_{m,j} - \Xi^A_{m,j}$, and in a region j (a weighted average across sectors).